

For each problem in this set you must submit both Algebraic and Graphical Solutions.

Note: The following is an example of a Diophantine problem (named for an Egyptian Mathematician named Diophantus)

1. The contents of a purse are not revealed to us, but we are told that there are exactly 6 pennies and at least 1 nickel and 1 dime. We are further told that if the number of dimes was changed to the number of nickels, the number of nickels was changed to the number of pennies, and the number of pennies was changed to the number of dimes, the sum would remain unchanged. Find the least possible and largest possible number of coins that the purse contains.

2. Find the smallest value of  $p^2$  for which the pair of equations has a solution other than  $x = y = 0$ , and find the ratio  $x : y$  for this value of  $p^2$

$$(4 - p^2)x + 2y = 0$$

$$2x + (7 - p^2)y = 0$$

3. Define a triangle by the following three equations:

$$7x - 12y = 42$$

$$7x + 20y = 98$$

$$21x + 12y = m$$

Find the value(s) of  $m$  for which the triangle has the largest and smallest areas.

4. Simplify the following  $\sqrt{1 + \sqrt{-3}} + \sqrt{1 - \sqrt{-3}}$

5. Find the shortest distance from the line  $3x + 4y = 25$  to the circle  $x^2 + y^2 = 6x - 8y$

6. Find the equation of a line with an x-intercept of 6 and y-intercept of -15.

7. Points  $A$  and  $B$  are on the parabola  $y = 4x^2 + 7x - 1$  and the origin is the midpoint of segment  $AB$ . What is the length of  $AB$ ?

8. Find the sum of the squares of the real roots of  $x^{256} - 256^{32}$

9. Find all **EXACT** (no decimal approximations) solutions (real and complex) to  $x^3 + 3x^2 + 3x + 7 = 0$

10. If the parabola  $y = ax^2 + bx + c$  passes through the points  $(-1, 12)$   $(0, 5)$   $(2, -3)$ , what is  $a + b + c$ . What is the equation of the circle passing through the points  $(-1, 12)$   $(0, 5)$   $(2, -3)$ ?