Problem Set #7 (Due Thursday, Oct 11th)

1. Two students attempted to solve a quadratic equation, $x^2 + bx + c = 0$.Although both students did the work correctly the first miscopied the 'b' term and obtained the solution set $\{-6, 1\}$. The second student miscopied the 'c' term and obtained the solution set $\{2, 3\}$. What is the correct solution set? (Note: a solution set is just the set of possible values for x)

2. What is the sum of the 8 eighth roots of 2 and the 12 twelveth roots of 1?

3. Both roots of the quadratic equation, $x^2 - 44x + k$ are primes. What is the sum of the possible values of k?

4. Find all solutions to the following equations: $3^{2x+2} - 3^{x+3} - 3^x + 3 = 0$ and $8^{x^2 + 3x + 10} = 4^{x^2 - x}$

5. If $9n^2 - 30n + c$ is a perfect square for all integers n, what is the value of c?

6. Use a substitution to solve $\left(x^2 - 3x\right)^2 - \left(3x^2 - 9x\right) = 4$

7. The product of three consectuive positive integers is 8 times their sum? What is the sum of the squares of those integers?

8. How many ordered pairs of integers (a, b) satisfy fall all of the following inequalities:

$$a^2 + b^2 < 16 \quad \text{AND} \quad a^2 + b^2 < 8a \quad \text{AND} \quad a^2 + b^2 < 8b$$

9. Find xy, given that $x^4 - y^4 = (-35 - 72I)$, $x^2 + y^2 = (-11 - 10I)$, and $x - y = (2 - 5I)$. (Recall: $I = \sqrt{-1}$)

(For Problem 10, Note you will wish to use the following formula for the area of a triangle with sides a,b,c: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where s is the semi-perimeter)

10. The lengths of the sides of a triangle are integers and the area is also an integer. One side is 13 and the perimeter is 32. What is the longest side? Find the median side of another triangle which has longest side 73 and a perimeter 150.