Problem Set #5

1. In Alice in Wonderland the following problem is found, "Let me see: four times five is twelve, and four times six is thirteen, and four times seven is--oh dear! I shall never get to twenty at that rate!" Mathematically demonstrate that the count never reaches 20.

2. Solve for x: $(x+1)(x+2)(x+3) = (x-3)(x+4)(x+5)$

3. Without using a calculator, show which is larger n **or** $\left(n^{\left(\frac{1}{2}\right)} + n^{\left(\frac{1}{3}\right)} + n^{\left(\frac{1}{4}\right)}\right)$ when n = 4 and n = 7?
Extra challenge, try it for n = 5 and n = 6.

4. When the repeating decimal, 0.481818181... is represented as a reduced fraction what is the absolute value of the difference of the numerator and denominator? What is the sum of the numerator and denominator of the reduced fraction which can be represented by the repeating decimal 2.52525252...?

5. What number is halfway between $\frac{1}{8}$ and $\frac{1}{10}$?

6. Which of the following numbers has the greatest value: 2^{201} **or** 4^{100} **or** 8^{70} **or** 16^{50} **or** 32^{39}?

7. Of the following sets of numbers: even integers, odd intgers, prime numbers, positive numbers, negative numbers, determine which are closed (or open) under addition and multiplication.

8. If N>1, then simplify $\sqrt[3]{N\sqrt[3]{N\sqrt[3]{N}}}$

9. Consider the following algebraic operation (defined for positive real numbers): $x \,\&\, y = \frac{xy}{x+y}$.
Determine whether "&" is Associative, Commutative, Both or Neither.

10. X, Y, **and** Z are disjoint sets of people (no person is in more than one group). The average ages of people in various sets is given below:

X: 37 Y: 23 Z: 41 X ∪Y: 29 X∪Z: 39.5 Y∪Z 33

What is the average age of people in the set $X \cup Y \cup Z$?