Advanced Algebra
Test #2

Open Book, Open Notes

$+1$ (out of 50) for each correct answer, $-\frac{1}{4}$ (out of 50) for each incorret answer

Section I

1. What is the complex conjugate of $5+8i$?

A. $5+8i$
B. $-5+8i$
C. $-5-8i$
D. $5-8i$ (correct)
E. None of These

2. Which is **not** one of the modes of mathematical representation

A. Verbal
B. Algebraic
C. Problem Solving (correct)
D. Numeric
E. Graphic

3. What is the degree of the polynomial $3x^5+2x^4+x^3+5x^2+10x+7$?

A. 7
B. 5
C. 3
D. 2
E. None of These (correct)

4. According to the blurb on page 97 on your textbook, which mathematician is responsible for the abstraction which lead to a solution formula to a general quadratic. (Yes this is a check to make sure you brought your book)

A. Blaise Pascal
B. Francois Viete (correct)
C. Leonhard Euler
D. Jean-Robert Argand
E. Gerolamo Cardano

5. This mathematician is the namesake of the plane on which complex numbers (with real numbers on the x-axis and imaginary numbers on the y-axis) can be visualized geometerically.

A. Blaise Pascal
B. Francois Viete
C. Leonhard Euler
D. Jean-Robert Argand (correct)
E. Gerolamo Cardano

6. Use the zero-product property to determine which of the following is **not** a factor of the polynomial, $x^4 + x^3 - 7x^2 - x + 6$. (Factoring is unnecessary and inefficient for this item)

A. $x + 3$
B. $x + 1$
C. x (correct)
D. $x - 1$
E. $x - 2$

7. (1987) $\left(1 + x^2\right)\left(1 - x^3\right)$ =

A. $1 - x^5$
B. $1 - x^6$
C. $1 + x^2 - x^3$
D. $1 + x^2 - x^3 - x^5$ (correct)
E. $1 + x^2 - x^3 - x^6$

8. (1954) The factors of $x^4 + 64$ are:

A. $\left(x^2 + 8\right)^2$
B. $\left(x^2 + 8\right)\left(x^2 - 8\right)$
C. $\left(x^2 + 2x + 4\right)\left(x^2 - 8x + 16\right)$
D. $\left(x^2 - 4x + 8\right)\left(x^2 - 4x - 8\right)$
E. $\left(x^2 - 4x + 8\right)\left(x^2 + 4x + 8\right)$ (correct)

9. (1992) Of the following which two have equal value?

I. $\sqrt{\sqrt[3]{4 \cdot 2}}$ II. $\sqrt[3]{2\sqrt{4}}$
III. $\sqrt[3]{4\sqrt{2}}$ IV. $\sqrt{2\sqrt[3]{4}}$

A. I and II B. I and III
C. II and III D. II and IV E. III and IV (correct)

10. If a is any postive integer, then which of the following is **not** a true statement

A. $2a + 1$ is always an odd integer
B. $\sqrt{a}$ is always a real number
C. $\sqrt{-a}$ is always an imaginary number
D. a^3 is always an odd integer (Correct)
E. The product of a and $\frac{1}{a}$ always equals 1

11. Which of the following is **not** equal to i^{21}?

A. i^{17}
B. i^9
C. i^{105}
D. i^{45}
E. i^{31}(correct)

12. Which of the following is the smallest positive number?

A. $10 - 3\sqrt{11}$
B. $3\sqrt{11} - 10$
C. $51 - 10\sqrt{26}$ (correct)
D. $10\sqrt{26} - 51$
E. Impossible to Determine without a calculator

13. Which of the following is the reciprocal of $8 - 3i$

A. $\frac{1}{8}$
B. $\frac{8 + 3i}{55}$
C. $\frac{8 - 3i}{55}$
D. $\frac{8 + 3i}{73}$ (correct)
E. $\frac{1}{11}$

14. (D) The discriminant of a quadratic equation with integer coefficients cannot be:

A. 23 (correct)
B. 24
C. 25
D. 28
E. 33

15. $\sqrt{5-12i}$ =

A. $2-2i$
B. $3-2i$ (correct)
C. $2+2i$
D. $3+2i$
E. $2+3i$

16. (1953) In solving a problem that reduces to a quadratic equation, one student makes a mistake only in the constant term of the equation and obtains 8 and 2 as roots. Another student makes a mistake on in the coefficient of the first degree term and finds -9 and -1 as roots. The correct original equation is:

A. $x^2-10x+9=0$ (Correct)
B. $x^2+10x+9=0$
C. $x^2-10x+16=0$
D. $x^2-8x-9=0$
E. None of these

17. (1979) The square of an integer is called a perfect square. If x is a perfect square then the next larger perfect square is:

A. $x+1$
B. x^2+1
C. x^2+2x+1
D. x^2+x
E. $x+2\sqrt{x}+1$ (Correct)

18. (1965) The number of real values satisfying the equation, $2^{2x^2-7x+5}=1$ is:

A. 0
B. 1
C. 2 (correct)
D. 4
E. More than 4

19. (1962) When $x^9 - x$ is factored completly into polynomials and monomials with real integer coefficients, the number of factors is:

A. more then 5
B. 5 (correct)
C. 4
D. 3
E. 2

20. Which value for c makes the polynomial, $49x^2 - 56x + c$, always equal to a perfect square

A. 784
B. -784
C. 16 (correct)
D. 8
E. The polynomial can never equal a perfect square

21. Factoring a polynomial to determine its roots corresponds graphically to finding its:

A. Y-Intercepts
B. X-Intercepts (correct)
C. Slope
D. Area
E. There is no corresponding graphical aspect to factoring a polynomial

22. What is the product of the three third roots of -1?

A. 0
B. 1
C. -1 (correct)
D. i
E. $-i$

23. What is the sum of the four fourth roots of 16?

A. 0 (correct)
B. $4 + 4i$
C. $4 - 4i$
D. $4i$
E. $-4i$

24. The n complex nth roots of a number can be used to:

A. Find the area of a circle
B. Construct a cube
C. Find the volume of a cylinder
D. Construct a Regular Polygon (correct)
E. There is no graphical significance to find complex roots

25. If $3^{5-x} = 81^{x+1}$, then $x =$

A. $\frac{1}{3}$

B. 3

C. $\frac{4}{5}$ (Correct)

D. $\frac{1}{2}$

E. $\frac{1}{5}$

26. If you were to solve the equation, $A = \frac{\pi d^2}{2} + \pi dh$ for d, then $d =$

A. It is impossible to solve for d

B. $d = \sqrt{\frac{2A - 2\pi h}{\pi}}$

C. $d = h - \sqrt{\frac{h^2 + A^2}{\pi}}$

D. $d = \sqrt{\frac{h^2 + A^2}{\pi}} - h$ (Correct)

E. $d = \sqrt{\frac{\pi}{2A - 2\pi h}}$

27. (1950) The real factors of $x^4 + 4$ are:

A. $\left(x^2 + 2\right)\left(x^2 + 2\right)$

B. $\left(x^2 + 2\right)\left(x^2 - 2\right)$

C. $x^2\left(x^2 + 2\right)$

D. $\left(x^2 - 2x + 2\right)\left(x^2 + 2x + 2\right)$ (correct)

E. None of these

28. If $2^3 \cdot 2^{3n} \cdot 2 = \frac{1}{4}$ then $n =$

A. $-\frac{5}{3}$

B. -2 (Correct)

C. $-\frac{2}{9}$

D. $-\frac{2}{3}$

E. $-\frac{1}{3}$

29. (1955) The numbers whose sum is 6 and the absolute value of whose difference is 8 are roots to the equation:

A. $x^2 - 6x + 7 = 0$
B. $x^2 - 6x - 7 = 0$ (Correct)
C. $x^2 + 6x - 8 = 0$
D. $x^2 - 6x + 8 = 0$
E. $x^2 + 6x - 7 = 0$

30. (1951) The expression, $21x^2 + ax + 21$, is to be factored into two linear prime binomial factors with integer coefficients. This can be done if *a* is:

A. any odd number
B. some odd number
C. any even number
D. some even number (correct)
E. zero

31. For which values of b are both roots of the polynomial, $x^2 + bx + 9$ imaginary?

A. $-4 < b < 4$
B. $-6 < b < 4$
C. $-4 < b < 6$
D. $-6 < b < 6$
E. None of these

32. The product of three consecutive numbers is 5 times their sum. What is the sum the squares of those numbers?

A. 50 (correct)
B. 110
C. 77
D. 29
E. None of these

33. Let the set of all values of *x* satisfying the inequalities $|x - 8| < 6$ and $|x - 3| > 5$ be written as $a < x < b$. What is $b - a$?

A. 6 (Correct)
B. 12
C. 14
D. 4
E. 16

34. A Heronian Triangle is a triangle whose area is an integer and whose three sides are all integers. The area of any triangle can be found using Heron's Formula, $A = \sqrt{s(s-a)(s-b)(s-c)}$ where s is the semi-perimeter. Which of the following sets is **not** the sides of a Heronian triangle?

A. $\{3, 4, 5\}$
B. $\{5, 5, 8\}$
C. $\{6, 8, 11\}$ (Correct)
D. $\{4, 13, 15\}$
E. $\{13, 14, 15\}$

35. Which of the following intervals is a solution to the inequality: $|3 - x| > 4$

A. $(-1, 7)$
B. $(-7, 1)$
C. $(1, 49)$
D. $(-\infty, -1) \cup (7, \infty)$ (Correct)
E. $(-\infty, -7) \cup (1, \infty)$

[BONUS] 36. (1963) The expression, $x^2 - y^2 - z^2 + 2yz + x + y - z$ has:

A. No linear factor with integer coefficients and integer exponents
B. The factor $-x + y + z$
C. The factor $x - y - z + 1$
D. The factor $x + y - z + 1$
E. The factor $x - y + z + 1$ (correct)

[BONUS] 37. (1944) Consider the equation $10z^2 - 3iz + k = 0$ where z is a complex variable (in the form $a + bi$) and $i = \sqrt{-1}$. Which of the following is true?

A. For all positive real numbers k, both roots are purely imaginary
B. For all negative real numbers k, both roots are purely imaginary (Correct)
C. For all purely imaginary numbers k, both roots are real and rational
D. For all purely imaginary numbers k, both roots are real and irrational
E. For all complex numbers k, neither root is real

[BONUS] 38.(1957) If $9^{x+2} = 240 + 9^x$, then $x =$

A. 0.1
B. 0.2
C. 0.3
D. 0.4
E. 0.5 (correct)