Geometry Expressions Manual

Saltire Software
PO Box 1565, Beaverton, OR

Contents

I. Solving Geometry Problems with *Geometry Expressions*

Geometry Expressions™ is an interactive application that lets you specify geometry problems with symbolic constraints — for example, set a line to have length *a*, set an angle to be θ, or make two lines perpendicular — rather than entering numeric values. The requested measurement is also output symbolically, as a mathematical expression.

After you've read about the basic ideas underlying Geometry Expressions, try running the *Geometry Expressions Tutorials* to help you get started.

Constraints and Constructions: Two Ways to Work

Use Geometry Expressions to solve problems with a straightforward three-step process:

1. Draw geometric objects such as points, lines, or circles. You need not draw them exactly — Geometry Expressions will make the necessary adjustments as you work.
2. Constrain their relationships. As you specify constraints, the drawing adjusts to satisfy them. You can constrain the problem fully, or you can leave some elements unspecified.
3. Request a measurement. Geometry Expressions adds any missing variables required for the calculation, then outputs the requested expression or value.

It's not important to draw objects exactly at first; instead, the application corrects the drawing one step at a time as you define the problem by adding constraints. This constraint-based approach enables a flexible, exploratory work style. It also readily lends itself to defining a problem symbolically, as it allows you to specify constraints — distances, angles, slopes, etc. — in terms of symbols. Geometry Expressions provides a rich set of geometry objects and constraints, described in the User Interface Reference and embedded Help system.

You may be familiar with other interactive geometry applications, such as Geometer's SketchPad® or Cabri Geometry™, which use a different approach:

1. Draw independent geometric objects.
2. Define dependent objects using constructions.

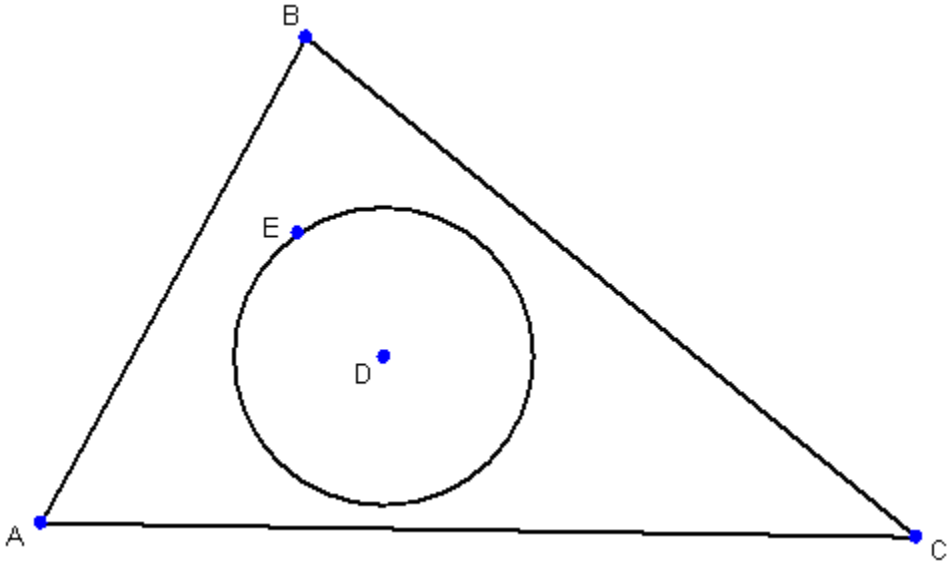
Geometry Expressions also enables this work style, providing a rich set of constructions, too (see the User Interface Reference and embedded Help system).

A ***construction*** defines a new geometry object in terms of existing objects. A construction-based approach requires you to distinguish between independent and dependent objects, and draw the independent objects accurately from the start. The drawing starts out correct and remains correct at each step. To enable this, the approach typically requires some foreknowledge of the problem geometry.

To compare these two approaches, we'll use both to create a drawing that shows the incircle of a triangle — the circle inside the triangle that's tangential to all three sides.

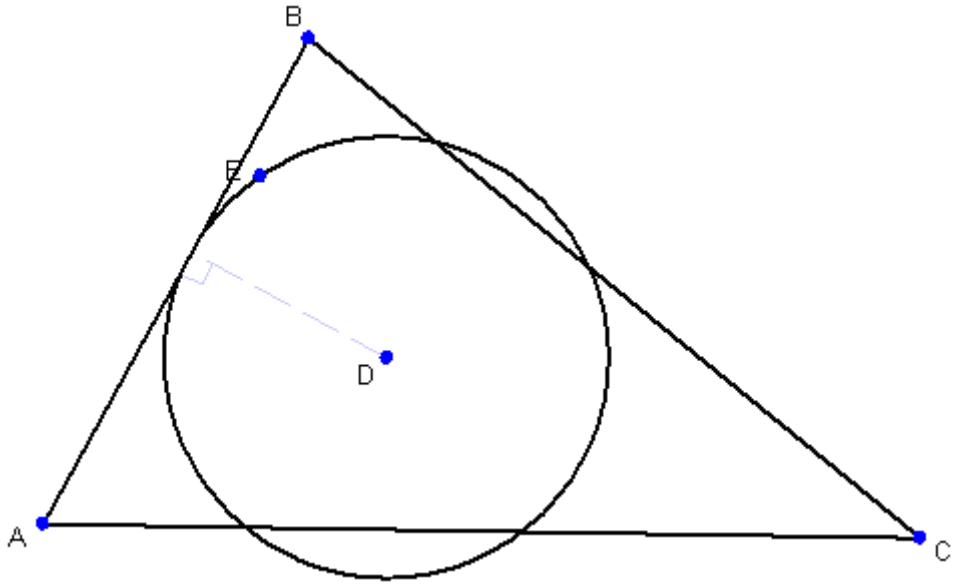
Constraint-based Drawing

A constraint-based approach starts with a drawing that contains all the pertinent geometrical objects, but does not at first require them to be correctly related (though the circle should lie more or less inside the triangle to distinguish it from the excircles):

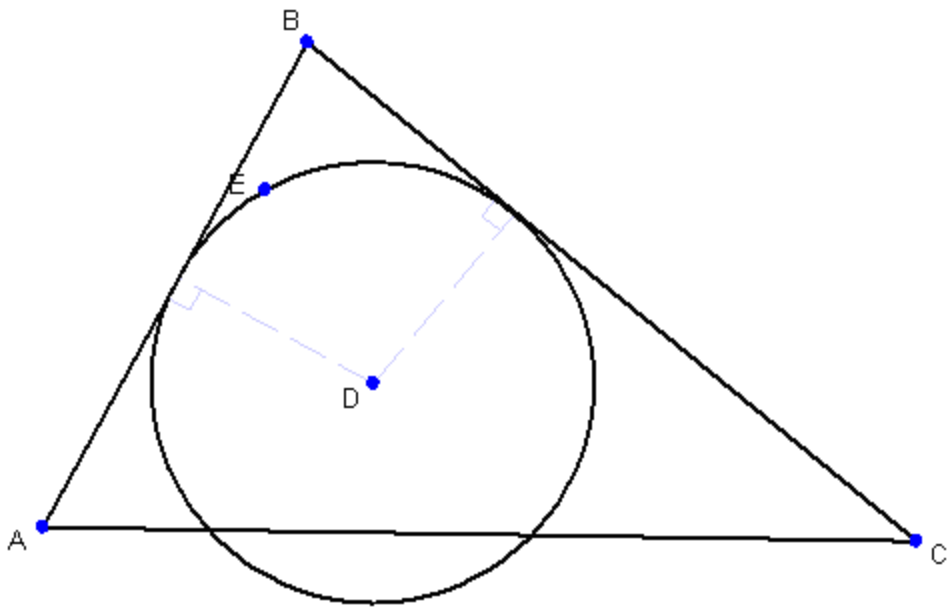


All we know of the incircle is that it is tangent to all three sides of the triangle. So we describe the problem by applying…

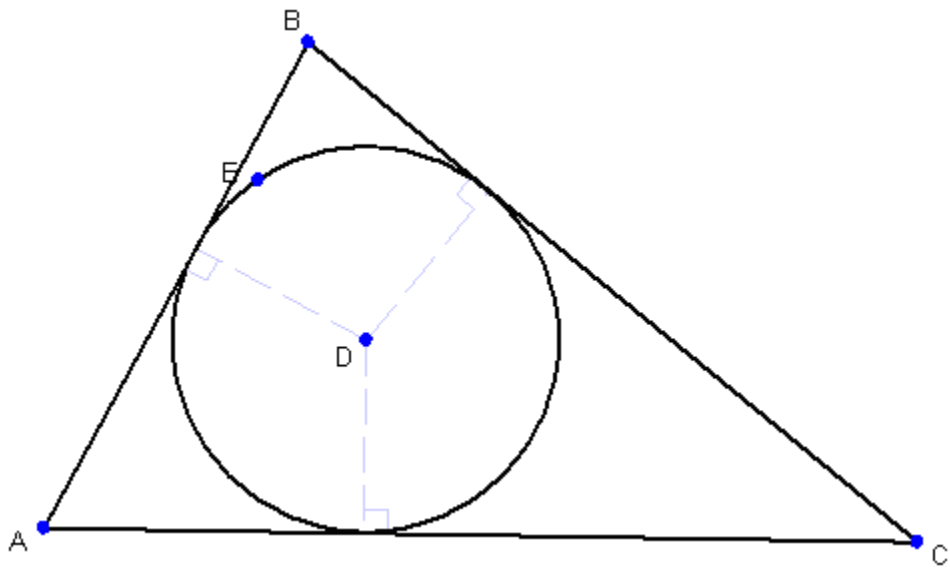
1. …the first tangency constraint:



2. …the second tangency constraint:



3. …and the last:

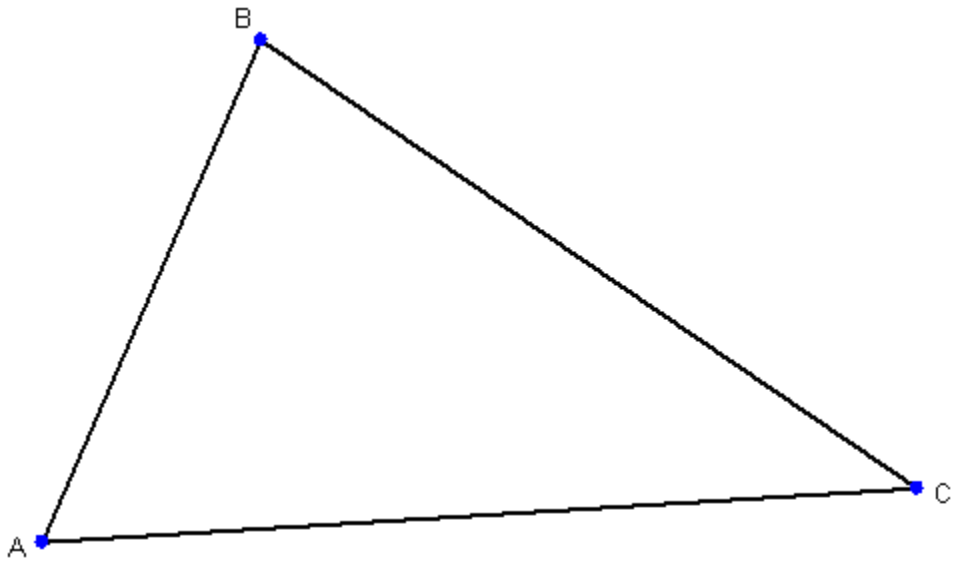


The drawing now accurately represents the desired situation.

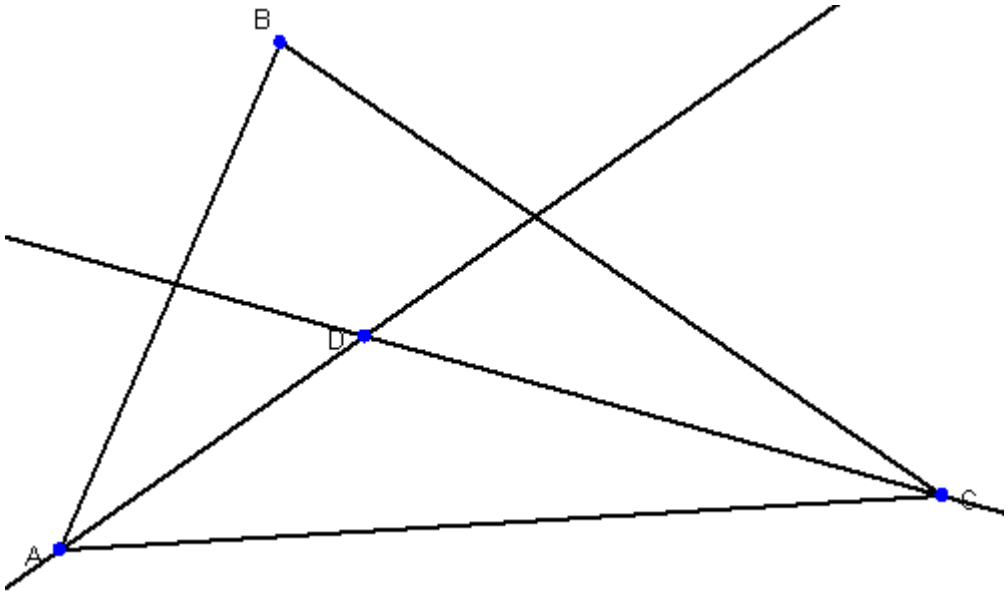
Construction-based Drawing

By contrast, a construction-based approach builds an accurate drawing step-by-step, relying on previous knowledge: that the center of an incircle is the intersection of two angle bisectors of the triangle.

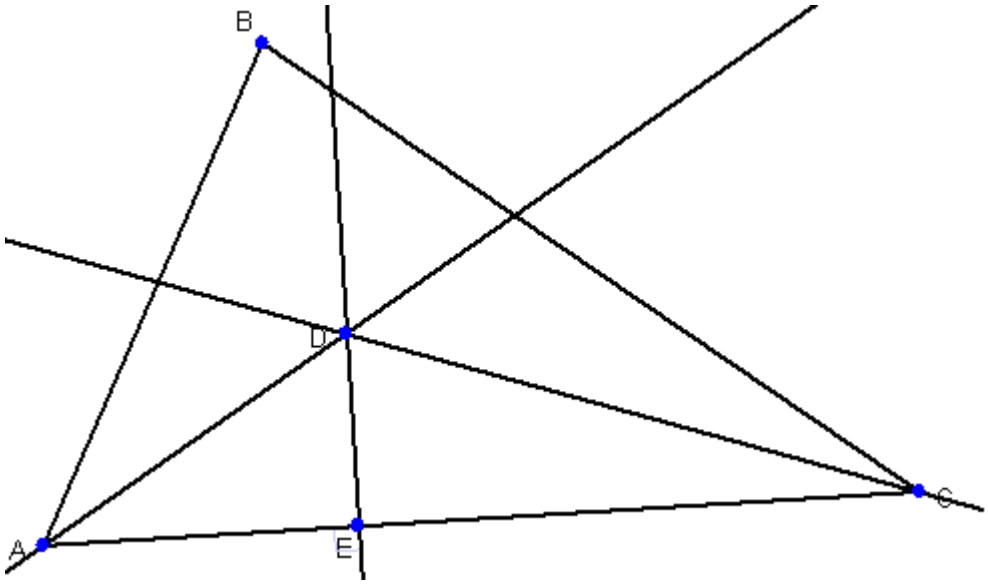
1. Draw the triangle:



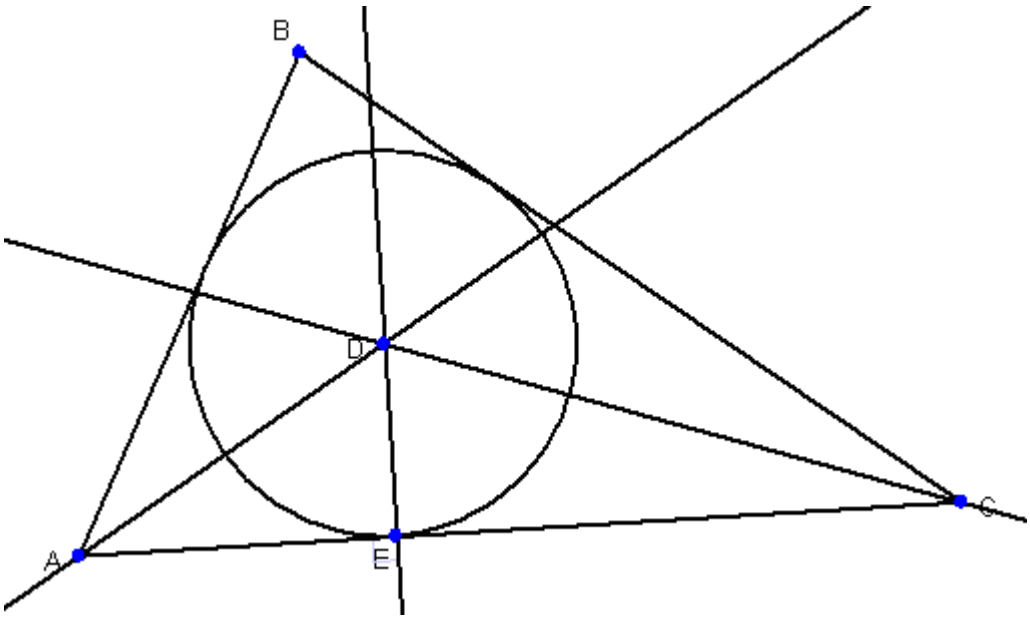
2. Construct the angle bisector of one angle.
3. Construct the angle bisector of the second angle.
4. Construct the intersection point D of the two angle bisectors:



5. Construct a line perpendicular to AC through D. Construct the point E, where the new line intersects AC:



6. Draw a circle centered at D which passes through E:



It requires some geometric foreknowledge or ingenuity to come up with this construction, nor is it clear without a proof that the circle is indeed tangent to sides AB and BC.

Construction-based drawing is the approach used by other interactive geometry applications, and therefore you may be used to it. Constraint-based drawing, on the other hand, enables a more natural, exploratory style of problem solving. In practice, you'll probably use a combination of these two approaches, making the geometry much easier to create.

How Does Geometry Expressions Solve Problems?

You can easily use Geometry Expressions without understanding anything about its internal process, but for various reasons, you'll find it helpful to understand how it builds the geometry problem and calculates the requested output. This understanding will make clear:

- how the application uses your drawings to resolve ambiguities;
- when and why the application adds variables;
- why it sometimes prompts you to resolve conflicts among constraints, and how to resolve these when they occur.

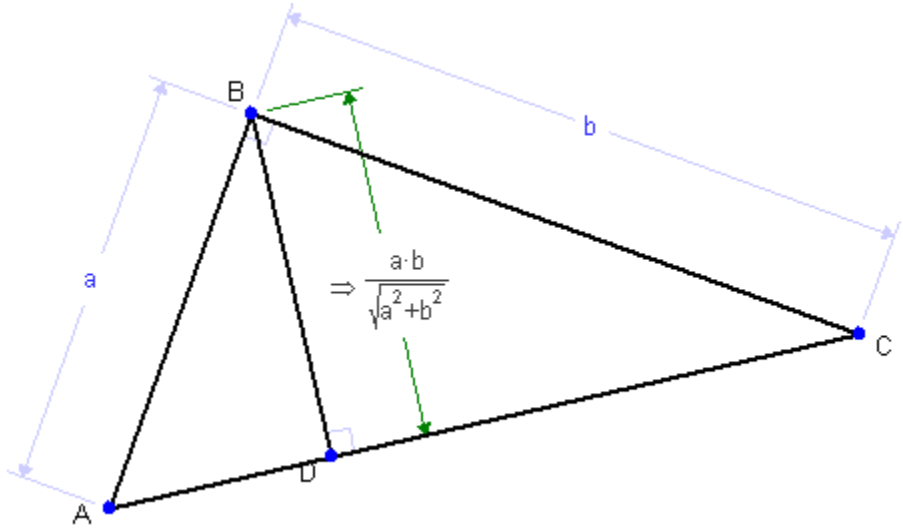
To shed light on these matters, we the offer following brief explanation of Geometry Expressions' internal process.

The Internal Construction Sequence

Though it's often natural and straightforward to describe a geometry problem in terms of constraints, internally, Geometry Expressions uses constructions.

1. As you specify geometric objects and constraints, it creates a construction sequence for them.
2. It executes those constructions to create a symbolic model of your geometry problem.
3. It creates a drawing that includes the objects and conforms to the constraints. To do so, it assigns sample numeric values for the variables.

For example, in the drawing below, AB is constrained to be length *a,* BC is constrained to be length *b,* AB is constrained to be perpendicular to BC, BD is constrained to be perpendicular to AC, and D is constrained to lie on AC.



Given this input, Geometry Expressions internally creates a construction sequence such as:

1. Place point A at an arbitrary location, creating variables to represent the coordinates of point A — by default, (u_0, v_0).
2. Put point B distance *a* from A in an arbitrary direction, creating another variable to represent the slope of line AB — by default, θ_0.
3. Calculate a sample value for *a* by measuring the distance of the line AB, using the native coordinate system. (Though the drawings we've shown so far do not show the axes, coordinates, or grid, you can reveal or hide them easily by clicking on the coordinate tool on the toolbar. See the User Interface Reference or embedded Help system.)
4. Create a line perpendicular to AB through B.
5. Put point C on this line at distance *b* from B.

   Two points are possible at this distance — either to the left or to the right of the line AB. Geometry Expressions takes its cue from where you drew the line segment and placed the point. If the drawing shows C to the right of AB, Geometry Expressions determines that's what you intended.
6. Find a line perpendicular to AC through B.
7. Put D at the intersection of this line and AC.

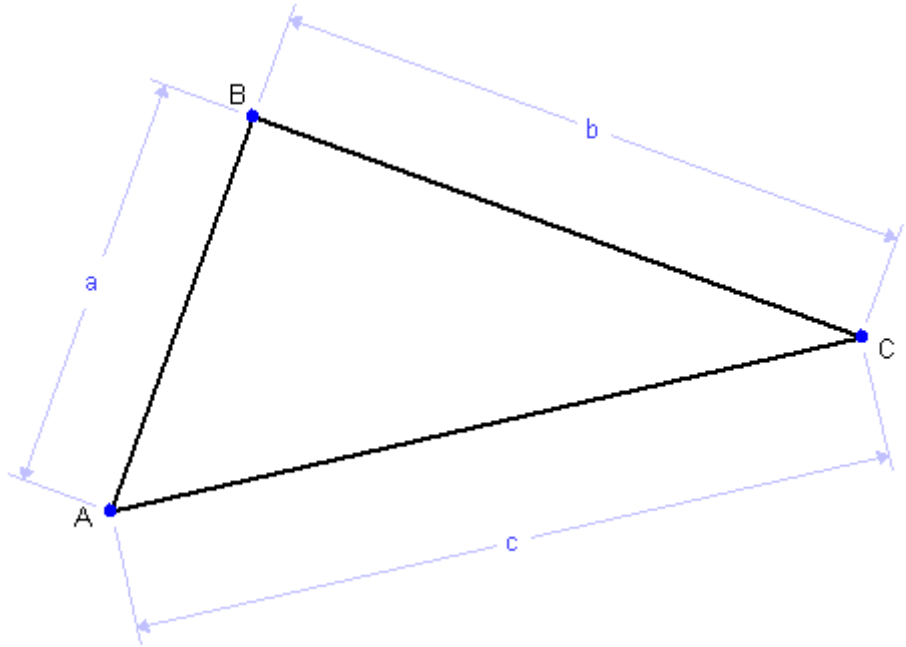
The existence of a construction sequence for Geometry Expressions is not equivalent to a problem being mathematically well defined. Unfortunately, Geometry Expressions' toolbox of constructions cannot cope with a number of mathematically well defined problems. Nevertheless, with a little ingenuity, you can usually find an alternative way to describe the problem, one that the application can construct.

Application-added variables, resolving geometrical ambiguities, and what happens when Geometry Expressions cannot find a construction sequence are all discussed in more detail below.

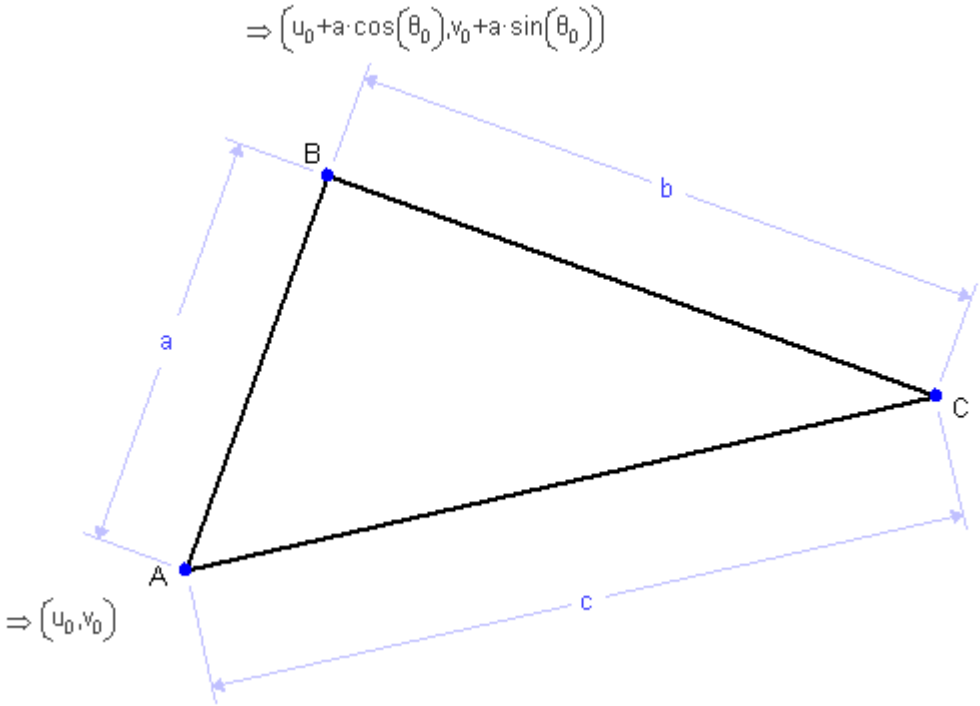
Application-added Variables

Geometry Expressions does not require you to specify your model in every detail. The underlying geometry engine automatically fills in any variables you've left unspecified.

For example, the following drawing (typical of many geometry problems) constrains the shape of the triangle but not its location:



Asking for the coordinates of points A and B, then, requires the system to add variables for the location of point A and the direction of line AB:

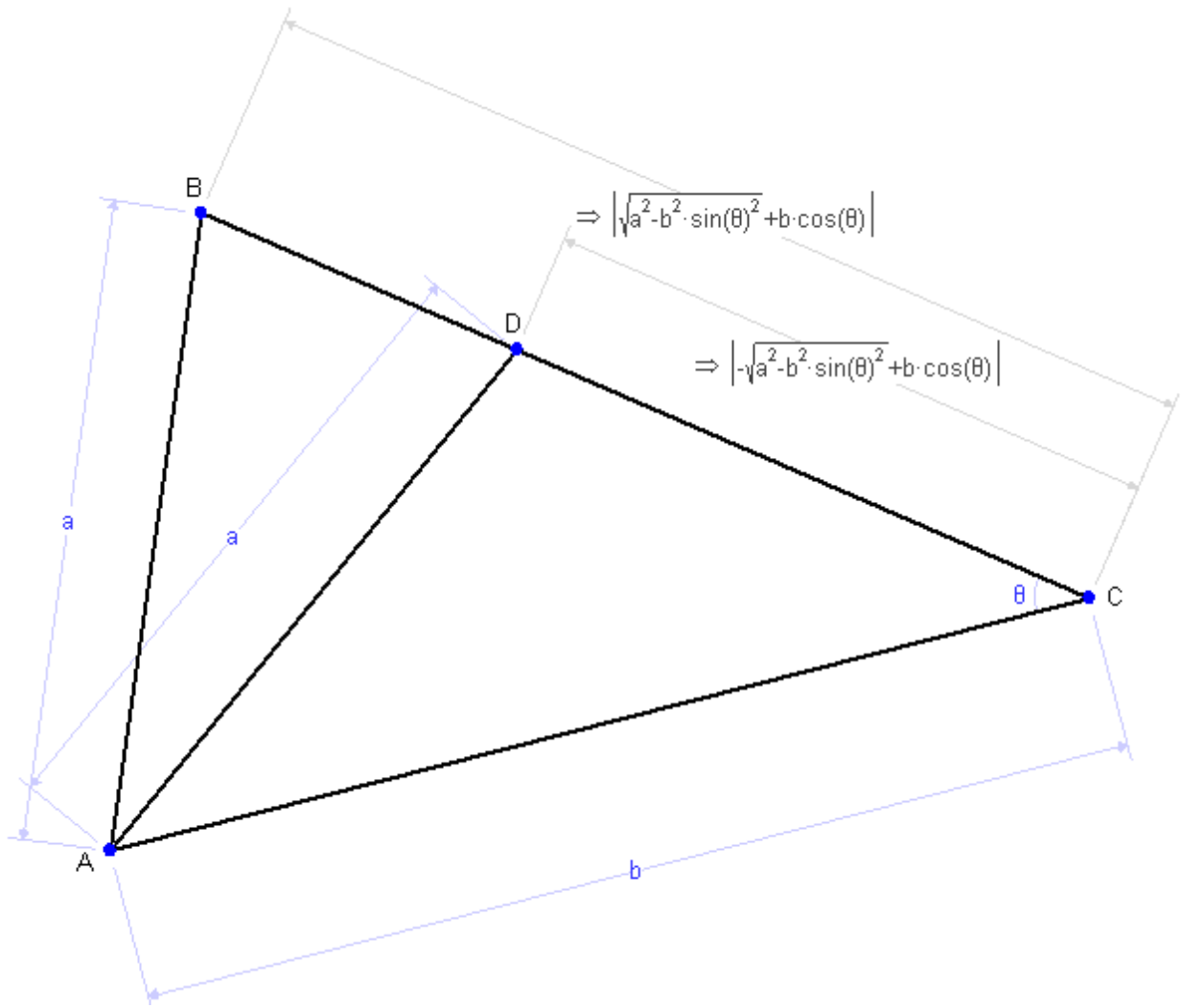


By default, Geometry Expressions creates the variables u_0 and v_0 for the *x* and *y* coordinates of point A, and θ_0 for the direction of the line AB. It associates numeric values with these variables based on the location of the pertinent object on the drawing's coordinate system.

Geometry Expressions cannot always calculate sample values so easily. When a variable is used in more than one place, or when it is involved in a complicated expression, the application solves equations to come up with appropriate sample values. However, whatever values it assigns, you can always set your own values directly in the Variables panel (discussed below, in "Controlling Variables", p. 28).

Resolving Ambiguity

Geometry Expressions uses your drawing to resolve ambiguities. For example, the following drawing shows two triangles, ABC and ADC. Both have two sides of constrained length, one of length *a* and the other of length *b,* as well as an angle θ, an unincluded angle:

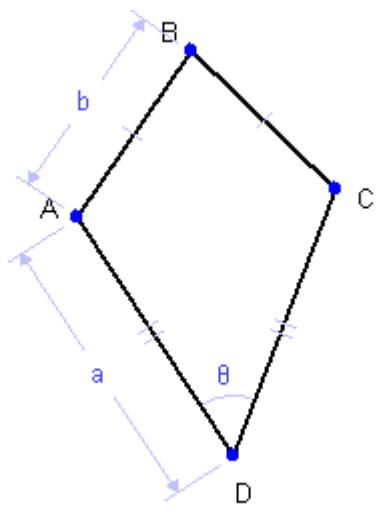


Two sides and the included angle uniquely define a triangle, but two sides and an unincluded angle do not. So if you specify a triangle using two sides and an unincluded angle, it's not unique: two possible solutions exist. In the drawing above, it could be either ABC or ADC.

In ambiguous cases like this, Geometry Expressions determines which solution you intend according to how you drew the objects.

Both triangles have one included angle(BAC and DAC), one unincluded yet constrained angle(ACB and ACD), and an unconstrained and unincluded angle (ABC and ADC). The key difference between the two triangles is whether this last angle is acute or obtuse. So the application consults your drawing to see how you drew this key angle. If you drew it as acute, the application constructs the triangle ABC; if you drew it as obtuse, it constructs ADC instead.

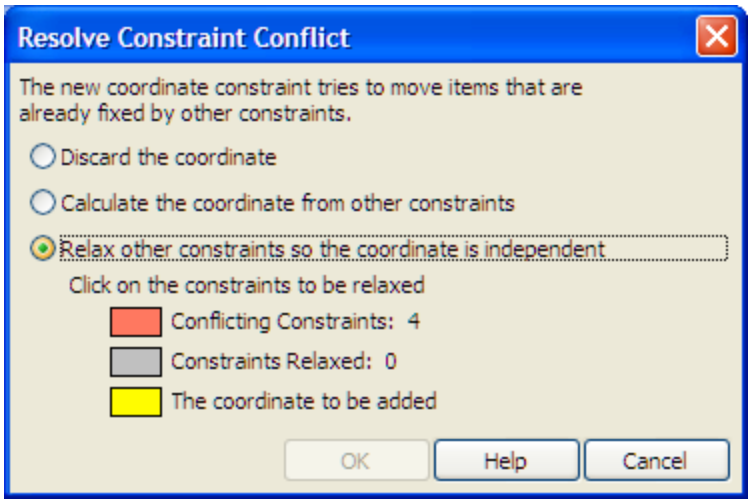
A trapezoid with the constraints specified below also has two possible solutions for the angle ABC — one convex, the other concave. Because the drawing is convex, Geometry Expressions chooses the convex solution:



Constraint Conflicts

You might try to add more constraints than necessary, creating a situation in which the constraint you're trying to add conflicts with existing ones. In such cases, Geometry Expressions cannot find a construction sequence.

If you enter a constraint for a geometry object that is already constrained, you'll see a dialog such as this:



You can resolve constraint conflicts in any of three ways:

- Cancel the operation to leave the drawing as it was without the new constraint.
- "Calculate *[new constraint]* from other constraints." Like canceling, this choice eliminates the new constraint. Unlike canceling, however, Geometry Expressions also calculates the selected geometry's value and displays the result.
- To add the new constraint, you must relax one of the conflicting ones. Choosing the bottom radio button affords the opportunity to choose which; all the conflicting constraints are highlighted in red, the constraint you tried to add is highlighted in yellow (*Figure 1*).

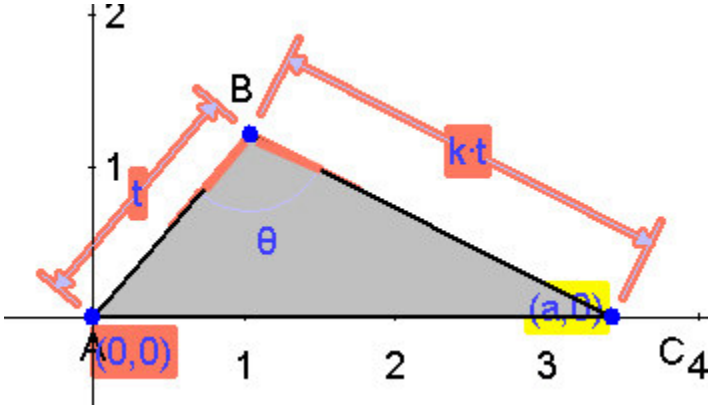


Figure 1

When you select one of these constraints (*Figure 2* shows the result of clicking on θ), the highlight of the current constraint changes to green.

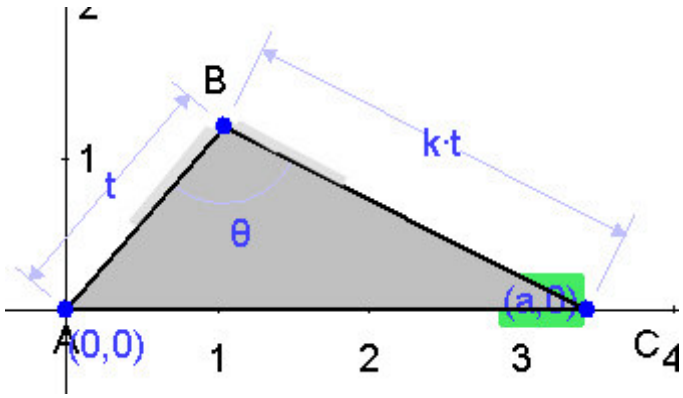


Figure 2

OK adds your new constraint, then calculates and displays the new value of the variable associated with the relaxed constraint (*Figure 3*).

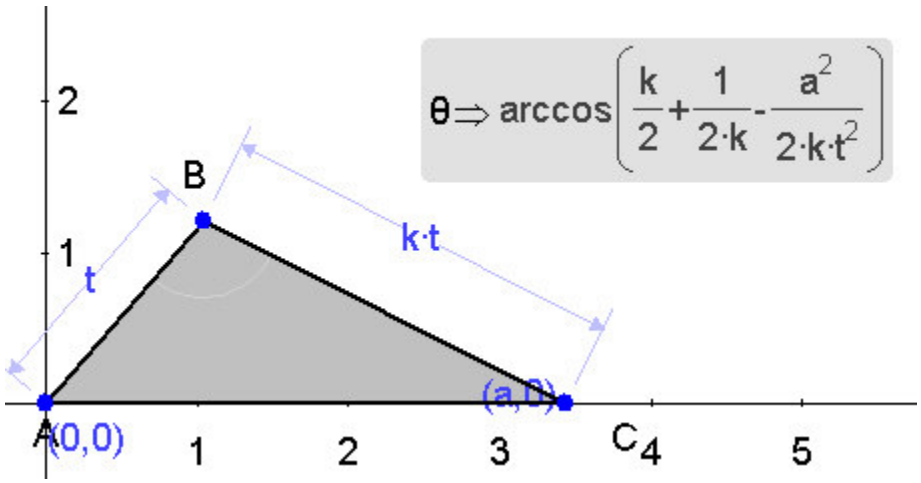
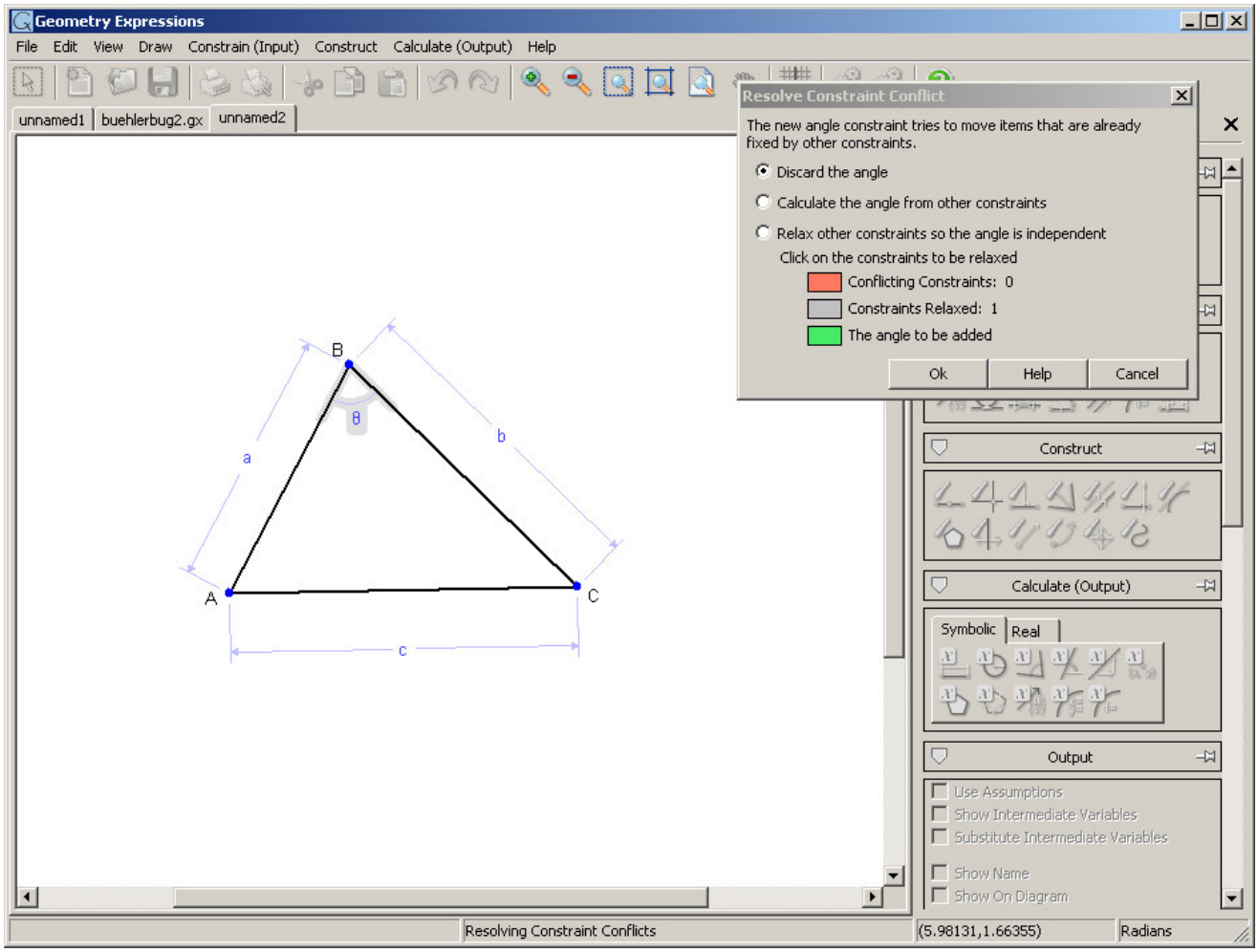


Figure 3

You'll need to resolve constraints conflicts whenever you add a new constraint that is:

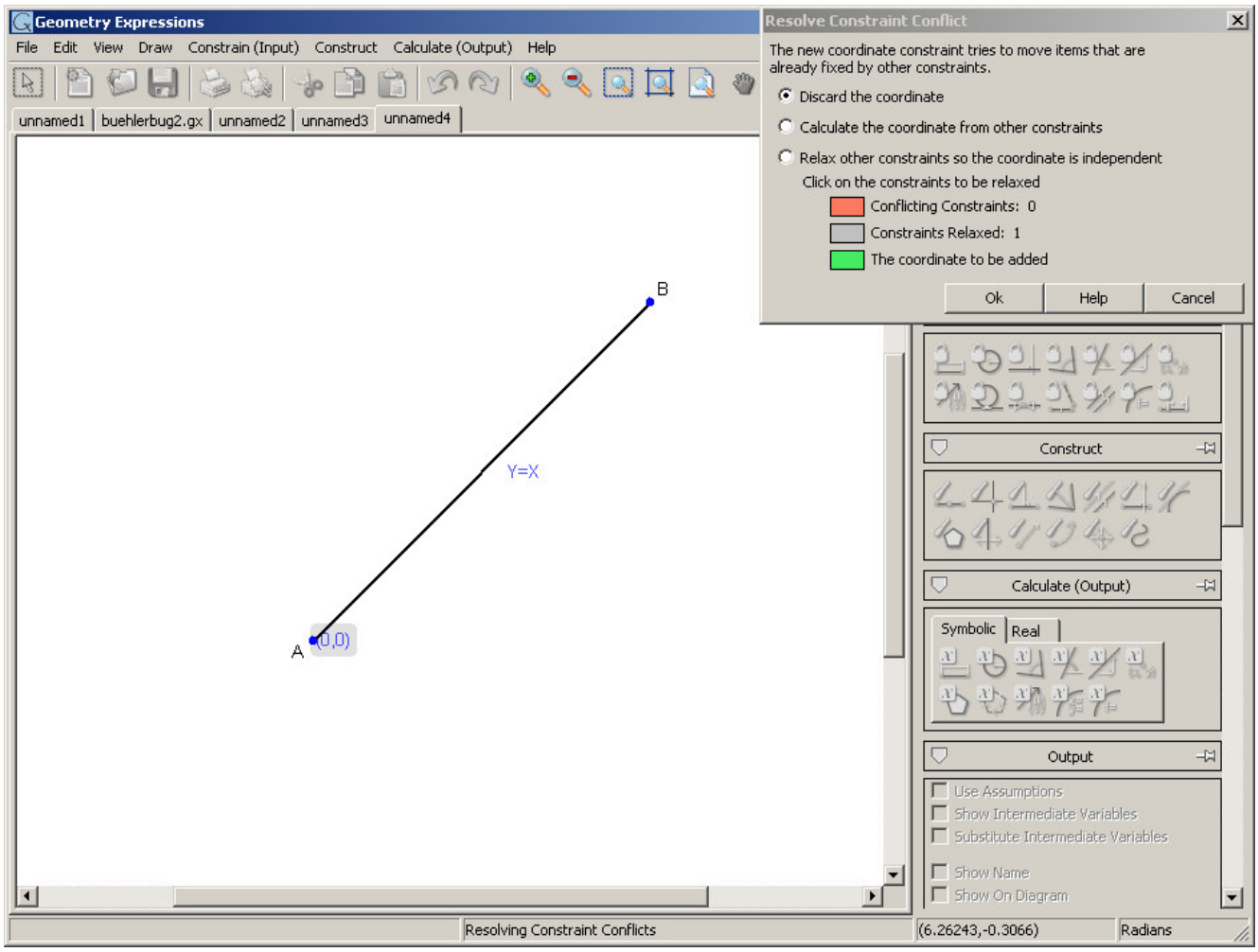
- not independent of those already added;
- not generically applicable, though possibly applicable in specific cases; or
- for which a construction sequence cannot be derived.

Below is an example of a constraint that isn't independent: three sides already define the triangle, so θ is dependent on the side lengths.

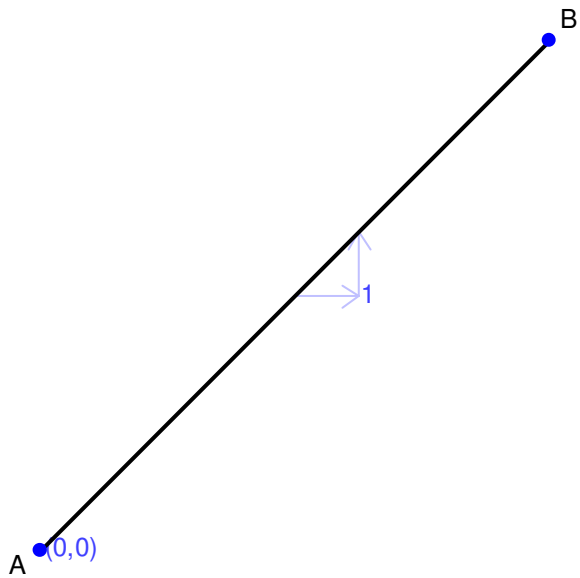


To resolve this conflict, you can eliminate one of the constraints — either one of the problematic side lengths, or θ.

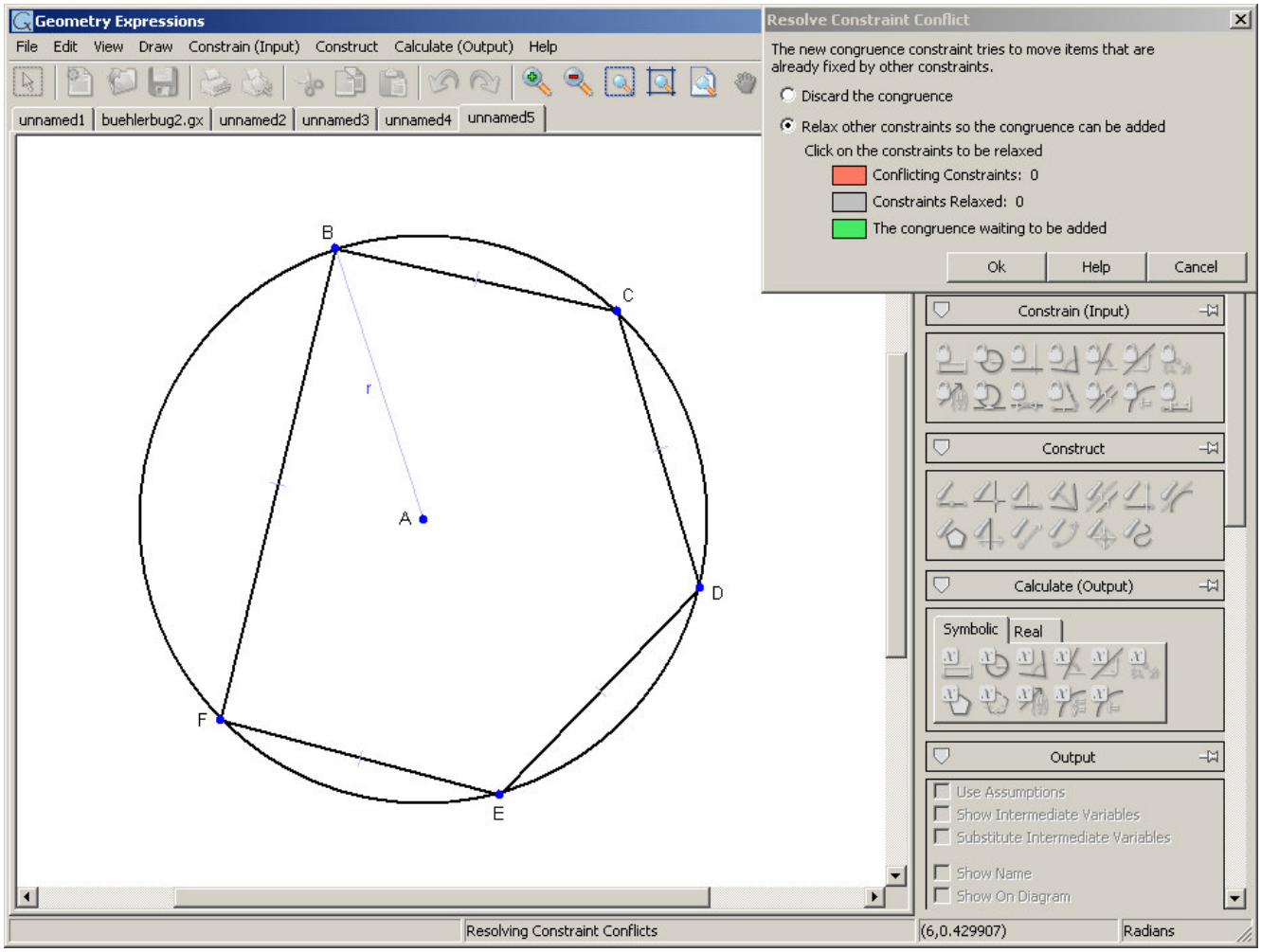
Below is an example of a constraint that, while it appears to be independent, would not be so in all cases. The line segment has been assigned the equation Y=X, and the point A has been assigned the coordinates (0,0). The point (0,0) does indeed lie on the line Y=X, but other values for its coordinates might not. For example, if you tried to constrain A to be at coordinates (1,0), the Y=X constraint would be violated. Constraints in Geometry Expressions must be applicable generically, rather than relying on specific values.



To resolve this conflict, instead of setting the equation of the line, you could set its slope:



Some problems seem to violate neither of these rules; nevertheless, Geometry Expressions can't find a construction sequence that satisfies them. For example, suppose you're trying to determine the length of the side of a regular pentagon inscribed in a circle of radius *r.* You might try to draw the pentagon by constraining its sides to be of equal length:



The constraints are certainly independent, so why the prompt to resolve a conflict? The answer lies in the details of how Geometry Expressions converts the constraint description into a construction sequence.

A construction sequence is a sequence of primitive geometrical operations, each of which creates a single geometric object. For example:

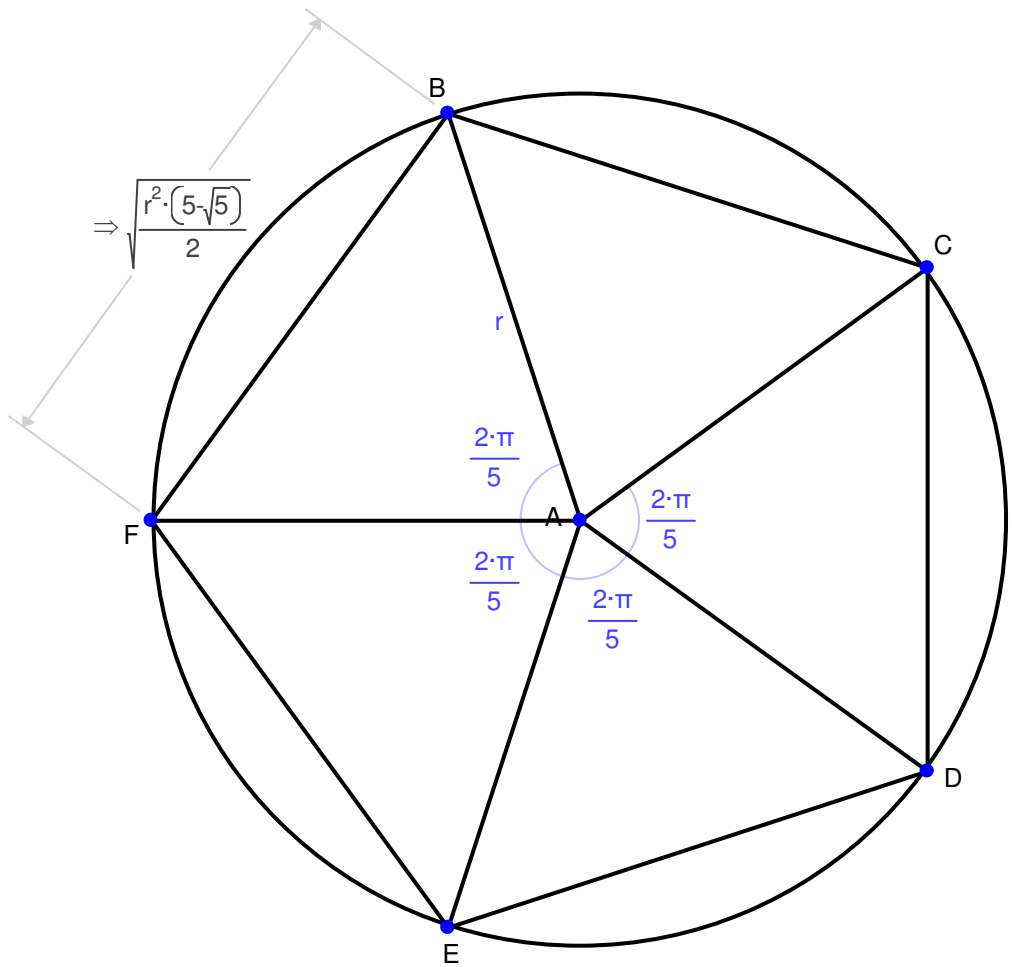
> Create a point on a given line, distance *a* from a given point.

or:

> Create a circle tangent to three given circles.

To draw the specified pentagon, Geometry Expressions starts with a circle of a particular radius. It then constructs a point on the circumference at an arbitrary location. Next, it must construct another point, this time one whose location is not arbitrary. But calculating the point's position would require simultaneously resolving all the congruent distance constraints. Geometry Expressions can’t resolve that many constraints at once; it can resolve them only in batches of two or three, with each batch defining just one geometrical object. So Geometry Expressions is unable to create the construction sequence.

To solve this problem, you need to try a different approach. For example, specify the angles from the center of the circle, which enables Geometry Expressions to construct the points one by one:



Calculating Output

Geometry Expressions calculates results based on the objects as you've drawn them, and the constraints or constructions you've specified. If you haven't supplied all of the necessary constraints, the system adds any missing variables automatically. (For an example, see "Application-added variables", p. 12.)

You can ask for calculations in either numeric or symbolic terms by choosing the appropriate tab **(Real** or **Symbolic**) when you request the output. Symbolic calculations yield results as mathematical expressions.

Geometry Expressions can present the results of its calculations in different ways, depending on your output settings. Two in particular require discussion —whether you wish to see absolute values for certain terms, and how you wish to treat intermediate variables.

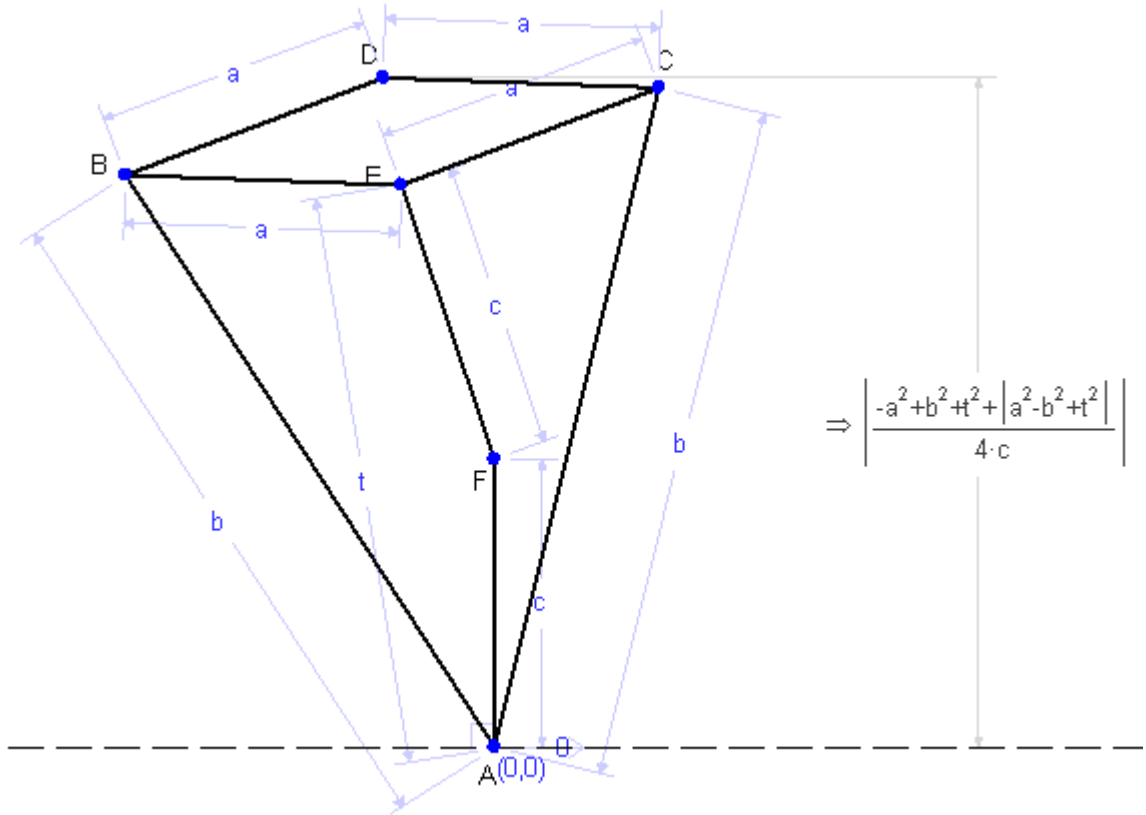
Eliminating Absolute Values

Use Assumptions is a checkbox in the Output panel that determines whether expressions include or eliminate absolute values: the assumption in question is whether the operand of the absolute value is positive or negative.

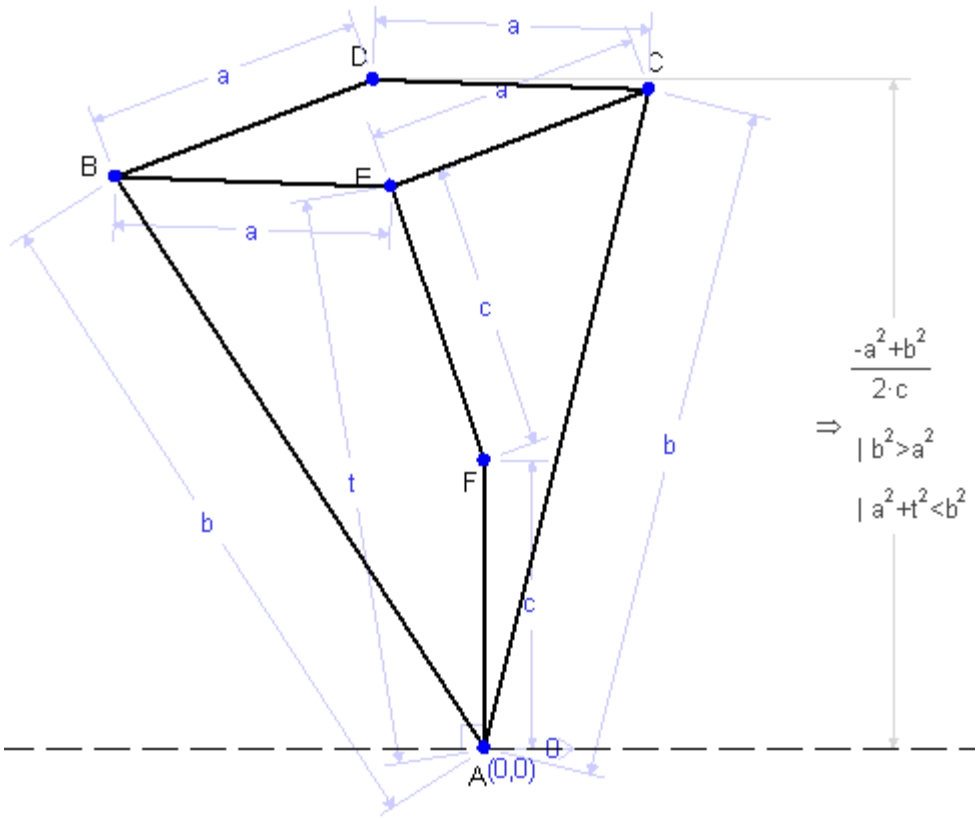
When checked, the application assumes that a given term is positive, if the value of the absolute value operand is positive. If it's negative, the corresponding term is assumed to be negative.

When unchecked, pertinent terms appear in expressions enclosed in absolute value bars.

For example, without assumptions, in this drawing of Paucellier's linkage, the height of D depends on parameter *t:*



If you check **Use Assumptions**, the expression simplifies and no longer involves *t.* However, other assumptions can sometimes be added, as shown below. Check these assumptions against the geometry of the diagram to see if they accurately express your intentions.



The second assumption in the drawing above is equivalent to assuming that angle AEB is obtuse; given the geometry of the drawing, is reasonable.

Assumptions are a means to simplify expressions and apply only to absolute values. If an expression doesn't include any terms in absolute values, the checkbox will have no effect.

Using Intermediate Variables

As described in "Calculating Output" above, the application expresses results in terms of:

- variables you've added, and
- additional variables the application added, if you haven't fully specified the drawing.

However, expressions can sometimes be so large and complex as to be difficult to read and understand. To make output expressions clearer and more compact, you can choose to use intermediate variables.

An *intermediate variable* is a variable defined in the same terms — your own variables plus those the system added — for the purpose of simplifying an expression and enhancing its readability. Not every expression needs or can benefit from intermediate variables, but for some expressions, their use allows one straightforward name to replace a significant portion of the expression.

You can control how Geometry Expressions handles intermediate variables in three ways:

- An **Intermediate Variable Complexity** setting determines how complex an intermediate variable must be before it appears in an expression.
- A **Use Intermediate Variables** checkbox determines whether, above this threshold, they appear or not.
- A **Show Intermediate Variables** checkbox affects their display.

These three settings interact with each other to let you control the calculations that Geometry Expression performs and how it presents them.

Intermediate Variable Complexity is an application setting, available from **Edit** > **Settings**. From the **Math** tab, this option allows you to set a threshold for intermediate variable complexity in terms of a number from 2 to 100. This number determines the complexity of the term to be replaced with a variable in an expression.

A low number tells the application to replace even simple terms with intermediate variables, so you'll see more of these variables. A progressively higher number means that intermediate variables will represent progressively larger and more complex terms in expressions, so you'll see fewer of them.

Use Intermediate Variables, a checkbox in the Output panel, determines whether intermediate variables above the complexity threshold are used or not. When checked, terms more complex than the threshold will be replaced by intermediate variables in expressions. Terms simpler than the threshold are not replaced.

When unchecked, intermediate variables never appear and the threshold has no effect.

Show Intermediate Variables, also a checkbox in the Output panel, determines whether intermediate variable definitions are displayed with results. When checked, you’ll see the results, including intermediate variables, followed by additional lines displaying the definition of each intermediate variable used.

When unchecked, you’ll see intermediate variables in expressions according to the other two settings, but not their definitions.

Show Intermediate Variables affects only how an expression is displayed; it has no effect on the underlying expression itself. And it’s relevant only when **Use Intermediate Variables** is checked. When **Use Intermediate Variables** is unchecked, there are no intermediate variables to display.

To summarize:

- **Intermediate Variable Complexity** sets a threshold above which intermediate variables are used to calculate expressions, and appear in them. They thus affect the details of the expression that's calculated, as well as affecting the expression's readability.
- **Use Intermediate Variables** determines whether to use intermediate variables above the threshold, or not. This also affects the details of expressions.
- **Show Intermediate Variables** determines whether to display the definitions of intermediate variables when they are present.

Controlling Variables

As you create geometry objects and add constraints or constructions, Geometry Expressions creates the required variables; assigns numeric values according to the constraints, constructions, and drawing; and lists any variables you create along with their values in the Variables panel.

The numeric value assigned to a variable is like a sample value; it does not replace the symbolic value — the expression. Instead, the application uses the numeric value for four purposes:

- to determine where to draw the geometric objects on the page,
- to resolve ambiguous geometry,
- to determine whether to substitute a positive or negative value for an absolute value when using assumptions, and
- to take numeric measurements from the drawing.

You don't have to accept the sample values assigned by the application. You can control the values of variables various ways:

- Assign a value directly.
- Lock a variable to its current value, so that it won't change when you drag geometry objects in the drawing.
- Assign a range of values to the variable for creating an animation.

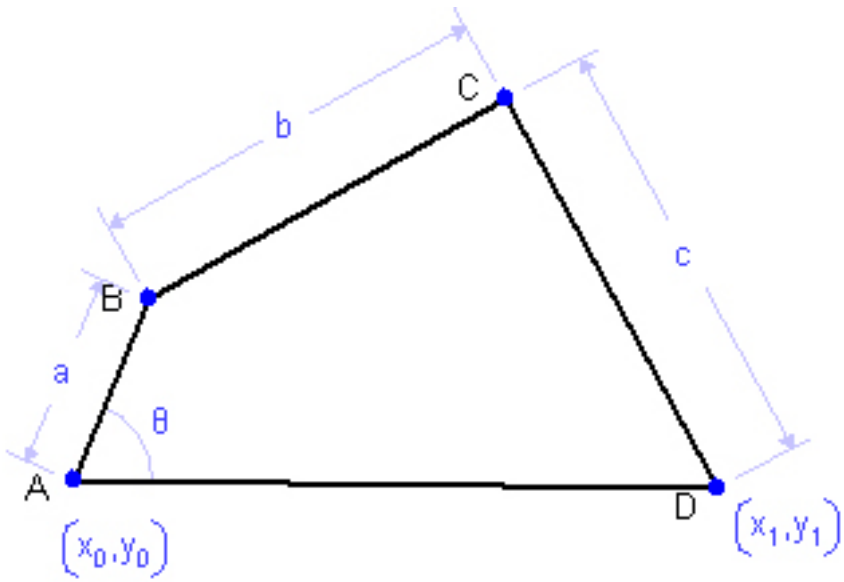
These capabilities are discussed below.

Assignment

If you wish a given variable to have a specific value, you can assign it directly by selecting the variable in the panel and entering the desired value in the input field. (See the User Interface Reference or embedded Help system.) If you assign a value that results in a geometry that the application cannot construct, the problematic objects in the drawing will disappear. However, they're not gone; they'll reappear when you set the variable to a value consistent with the rest of the drawing.

Locking

By default, when you drag points in a Geometry Expressions model, it adjusts the variable values to accommodate the drag as best it can. For example, in the model of a four-bar linkage below, dragging point B changes lengths *a* and *b* and angle θ:



However, you may wish the drag to act as if lines AB, BC, and CD were rigid, and only angle θ adjustable. To arrange this, lock all the variables except θ.

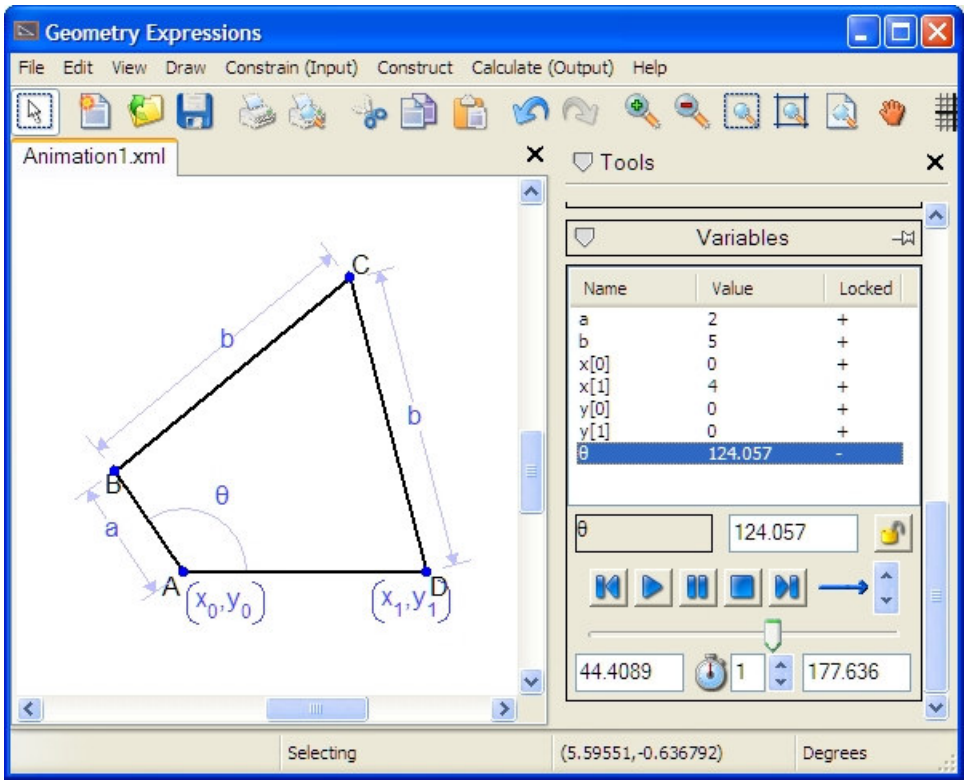
Locking a variable applies only to its behavior when you drag the drawing. You can still assign it a value by entering it in the input field.

Animation

Geometry Expressions allows you to set a range of values to a variable, then run through the range to animate your drawing, using the familiar video playback interface. To animate a drawing:

1. Select a variable to drive the animation.
2. Assign a range of values to that variable.
3. Press **Play**.

In the figure below we select θ to crank the linkage:

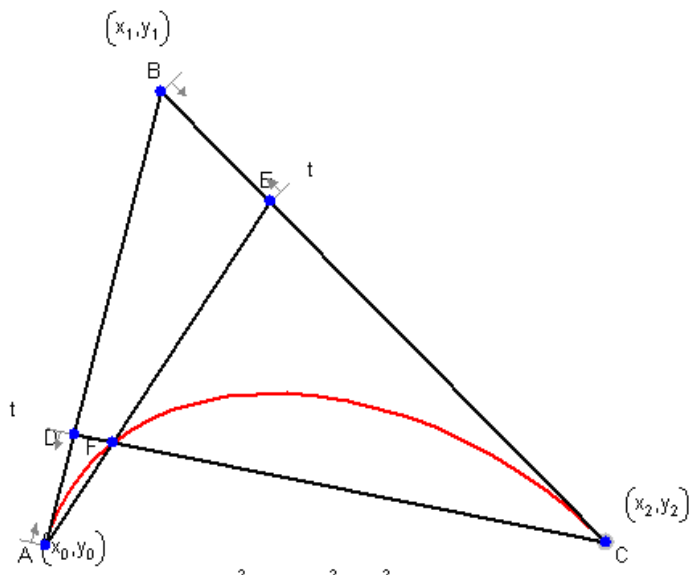


From other interactive geometry applications, you may be familiar with animating points along line segments or curves. Geometry Expressions provides this animation capability as well, using the constraint *point proportional along a curve*.

A point proportion *t* along a curve is defined thus:

for:	***definition:***
line segment AB	point (1-*t*) A + *t*B
circle	point on the circle that subtends angle *t* at the center
locus or envelope	point at variable value *t*

For example, in the following drawing, D is defined proportion *t* along AB, and E is defined proportion *t* along BC. The curve is the locus of F as *t* varies between 0 and 1:

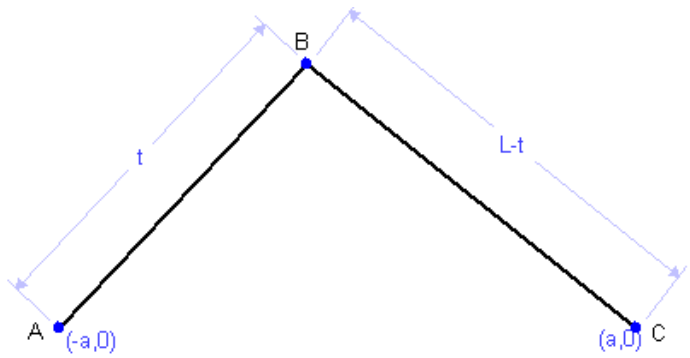


$$\Rightarrow x=\frac{x_0-2\cdot t\cdot x_0+t^2\cdot x_0+t\cdot x_1-t^2\cdot x_1+t^2\cdot x_2}{1-t+t^2}$$

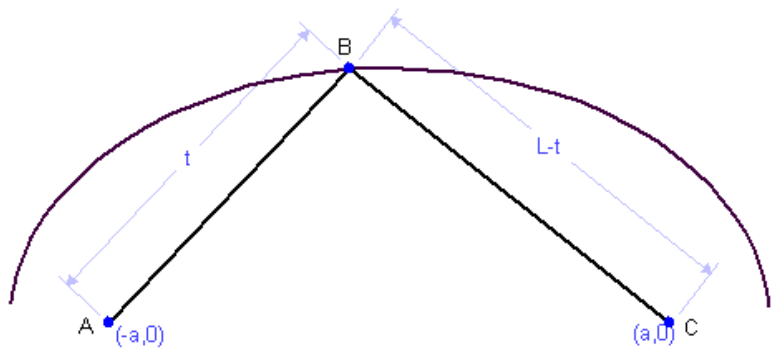
$$\Rightarrow y=\frac{y_0-2\cdot t\cdot y_0+t^2\cdot y_0+t\cdot y_1-t^2\cdot y_1+t^2\cdot y_2}{1-t+t^2}$$

To animate this, we assign *t* the range 0 to 1, then push **Play**.

The construction of both locus and envelope curves can be defined in terms of any variable. For example, starting with the drawing below:



…we create a locus over values of *t,* keeping other variables constant:



Importing and Exporting Expressions

You can copy expressions from Geometry Expressions and paste them into an algebra application such as Maple™ or Mathematica®, and you can also copy expressions from such applications and paste them into Geometry Expressions.

To enable this, Geometry Expressions uses the standard MathML™ interchange format. MathML — Mathematical Markup Language — is an extension of the widely adopted XML protocol, designed for mathematics applications.

The official MathML web page and specification is at:

www.w3.org/Math/

MathML comes in two varieties:

- Presentation MathML
- Content MathML

When copying from a companion application into Geometry Expressions, use Content MathML, which is the variety that Geometry Expressions accepts.

As a convenience, Geometry Expressions places expressions you copy on the clipboard as both Presentation MathML and Content MathML. Many applications can accept this dual input and choose the variety they require.

Some applications, however, might not be able to accept dual input. If you are having difficulty with an expression copied from Geometry Expressions:

1. Consult the user's guide or help system for the target application to determine which variety of MathML it accepts.
2. Use **Edit > Copy As…** to invoke a dialog enabling you to choose between the two varieties. Only the one you choose is placed on the clipboard.

Performance Tips

Various factors affect the simplicity and readability of the result and also, in some cases, the speed at which it is reached:

- How clearly have you defined your problem? The fewer unknowns you use, the faster the calculation and the more likely that the resulting expression will be simple and clear.
- How constrained is the problem? Fully constrained problems produce better results than underconstrained ones, because you control which unknowns are used.
- Using intermediate variables makes expressions less likely to simplify, but more likely to produce a result.
- For faster calculations, use lower settings of **Intermediate Value Complexity**.
- Use assumptions for simpler results, if your expression has absolute values in it. But check the assumption to make sure it's what you intend; if not, change the drawing to reflect your intention more accurately.

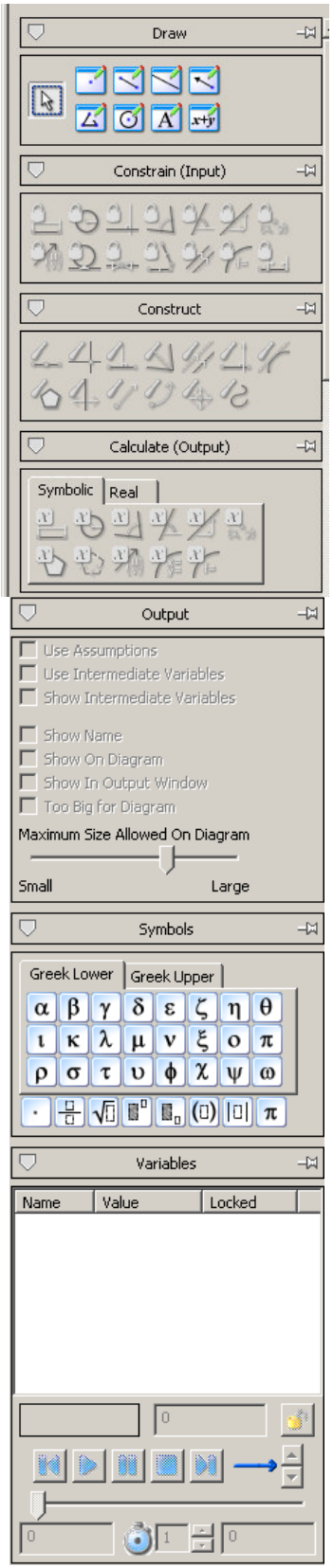
II. *Geometry Expressions* Tutorials

These tutorials are provided to help you get started using *Geometry Expressions.* They refer to the ideas discussed in *Solving Geometry Problems Using* Geometry Expressions, which we recommend you read before proceeding with this tutorial.

Documentation Conventions

You see:	It means:
Enter	the Enter key
Control-Z	Press ***Control*** and ***Z*** at the same time.
Enter	Press ***Enter*** (This is usually done as the final step in completing some desired action.)
File > New	Execute the **New** command from the **File** menu.

Tutorial 1: Define and solve a problem.



In this tutorial, you will:

— explore the tools,

— define a simple problem,

— calculate the solution,

— resolve a constraint conflict, and

— add and calculate an expression.

As you work, remember that there's no need to draw lengths and angles accurately. Just draw a sketch. Then, as you enter constraints and constructions, Geometry Expressions will update your sketch accordingly.

If you do happen to make a mistake, Geometry Expressions has multiple levels of **Edit > Undo** (Control-Z) and **Edit > Redo** (Control-Y) so that you can go back and forth through a sequence of steps.

Explore the user interface.

1. Start Geometry Expressions
2. **File > New**

   A new blank page opens with the tool palettes on the right:

   — **Draw** to create geometry objects,

   — **Constrain (Input)** to input any constraints,

   — **Construct** to create common, often used, geometry objects and

   — **Calculate (Output)** to make calculations associated with selected geometry elements.

   Below these are palettes that provide ways to:

   — control the way expressions are calculated,

   — insert symbols, and

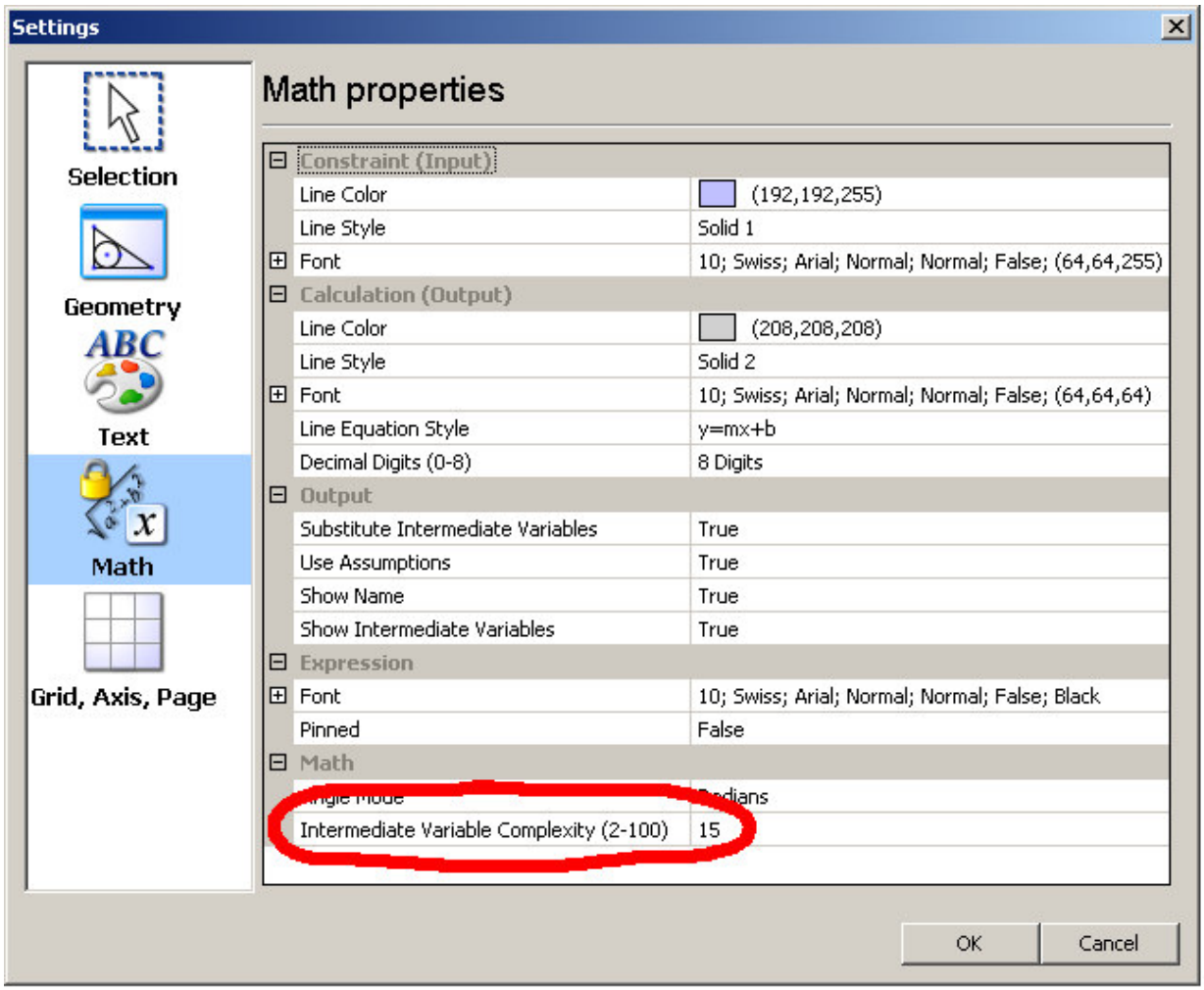
   — assign variable values.

   Depending on your display size, you may need to scroll to see them all.
3. You can open or close each palette individually by clicking the arrow at the left of the palette title. In this way, you can close palettes that you won't be using. These tutorials won't be using

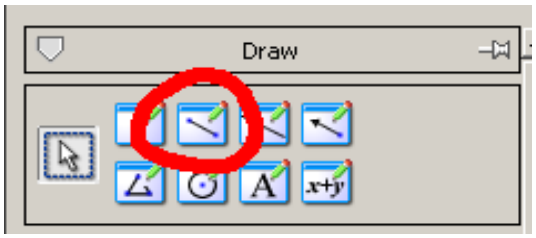
the Symbols palette, so you may close it now, if you wish.

You can customize the user interface in various ways. The options available in the **View** menu control zooming, scaling, axes etc. and the items in **Edit > Settings**, address other visual, text and mathematical properties.

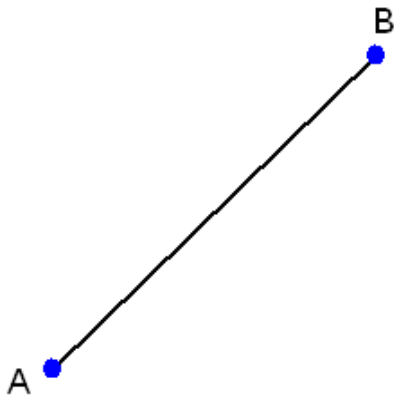
Under **Math**, for example, you can set the threshold for Intermediate Variable Complexity. A low threshold tells the application to substitute only simple terms with intermediate variables. The more complicated ones will not be substituted. A progressively higher threshold results in the automatic substitution of progressively larger and more complex terms with intermediate variables.



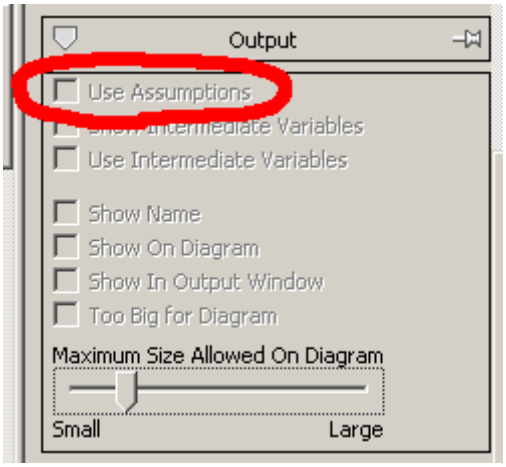
4. In the Draw palette, choose the line segment tool.



5. Starting near the bottom left of the drawing area, click and hold the mouse button down as you draw a line segment going upward and to the right. Now release the mouse button. This will complete the drawing of a line segment. Geometry Expressions places the first point at the beginning of the line (where the cursor is when you first press the mouse button) and the second point at the end of the line segment (where you released the mouse button). Another way of drawing a line segment is to click over the start point and move the cursor to the position of the end point and click again. (*I.e.* it's not necessary to hold the mouse button down while positioning the cursor.) The end points are labeled A and B respectively.

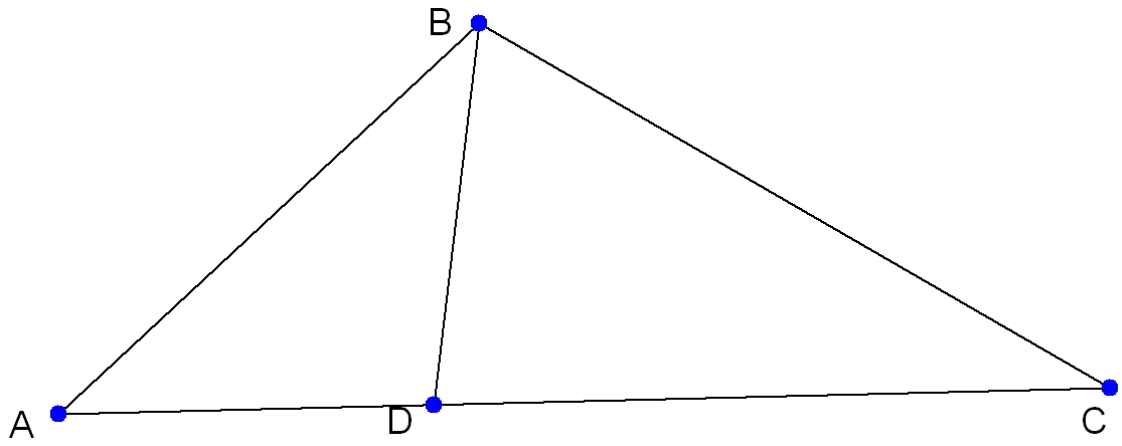


6. With the line segment tool still selected, move the mouse around the drawing area, passing over one or both of these points. Geometry Expressions tracks the location of the drawing cursor, snapping it to an existing geometry object when the cursor gets close enough.
7. Try creating other kinds of geometry objects using some of the other drawing tools. When you're ready to go on, undo them all except the line AB.
8. Go to the Output palette. (Scroll down to find it, if it is not already visible.) This is where you will see a variety of options for controlling the output answers to your geometrical questions. An output expression will have to be highlighted before you will be able to select any of the options in this palette. For example, if the **Use Assumptions** checkbox is selected, then Geometry Expression will assume that symbolic lengths will be positive. This assumption often simplifies the output.



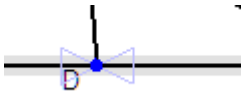
Define a problem.

To start, we'll use the first line segment and add three more to create something like this:

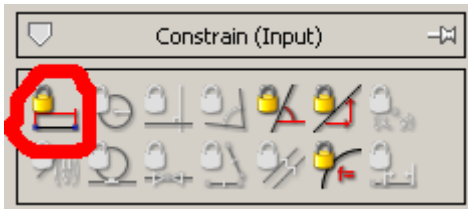


1. With the line segment tool selected, place the cursor near point B and draw three more line segments. (Remember, there's no need to draw precisely.)

   To ensure that point D lies along the line AC, move the cursor near to AC until the line becomes selected, D will then snap to this line.

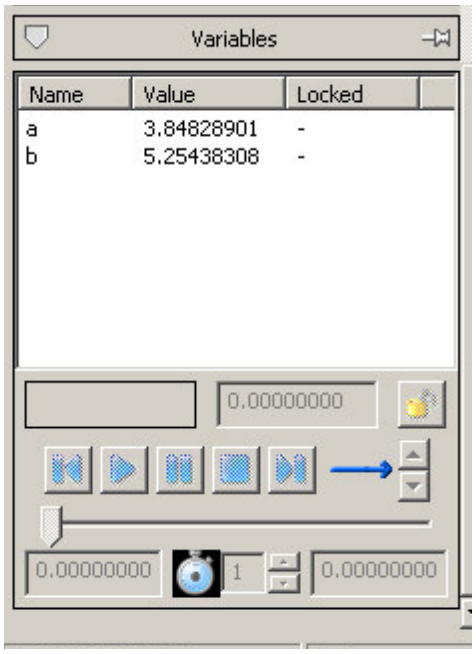


2. Change to the selection tool and select line AB by clicking the mouse anywhere along its length.
3. With one object selected, some of the tools in the Constrain (Input) palette are enabled. Click the Distance/Length constraint. If a tool is not available its icon will be grayed.



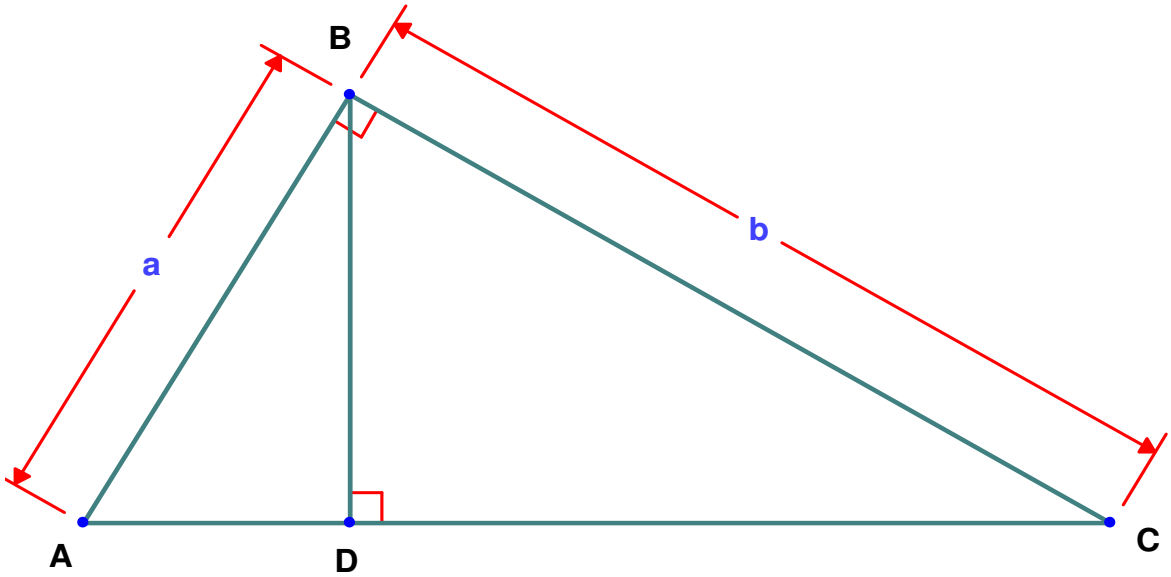
The constraint appears, along with the default variable name of *a,* selected and ready to change. Press ***Enter*** to accept it.

4. Select line BC and repeat the previous two steps to constrain its length to *b.*
5. You've now added two variables to your drawing. Go to the Variables palette (scroll down, if necessary) to see their names and values. The values displayed in the Variables palette are the current values, as taken from the default coordinate system. Notice that these values will change as you vary the lengths of the sides. Vice versa, if you select a variable and change its value in the Variables palette, the geometry will change accordingly.



6. Select the variable *a* in the Variables list and, in the input field below, round the value to the nearest whole number.
7. Select both lines AB and BC. (You can select more than one object at a time by making the first selection, then holding down ***Shift*** as you make subsequent selections.)
8. With two objects selected, you'll see a different set of constraint tools enabled. Click the Perpendicular constraint to constrain the angle to 90º. ABC is now a right triangle.

   Notice that point C moved, not point A, though either could have moved to satisfy the constraint. When it can, Geometry Expressions moves the most recently added object.
9. There's one last constraint to add. Select both lines BD and AC, and constrain their angle to be perpendicular as well.

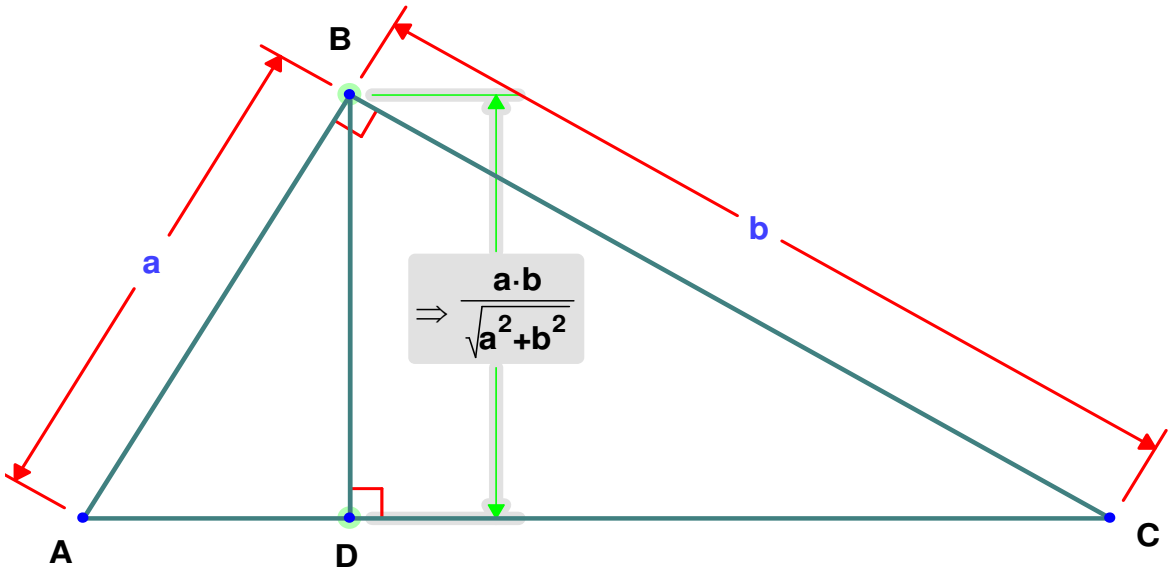


Calculate output.

Suppose you want to know the length of the line BD:

1. Select BD.
2. The Calculate (Output) palette offers output as both numbers — **Real** — and expressions — **Symbolic**. Click on **Symbolic,** if it's not already in front, then click the length icon.

An expression appears, showing the requested length in terms of the two variables, *a* and *b.* If you select this expression, you will be able to move it wherever you wish.

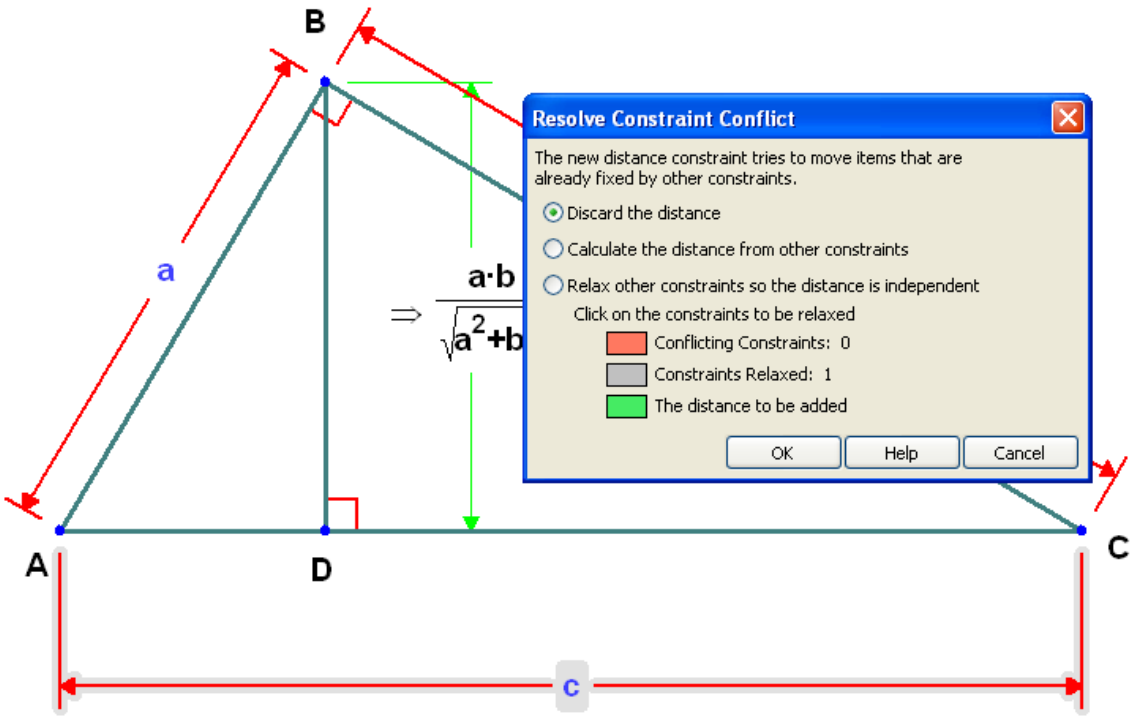


The double arrow => that appears to the left of the expression indicates that the expression is an output that the application has calculated.

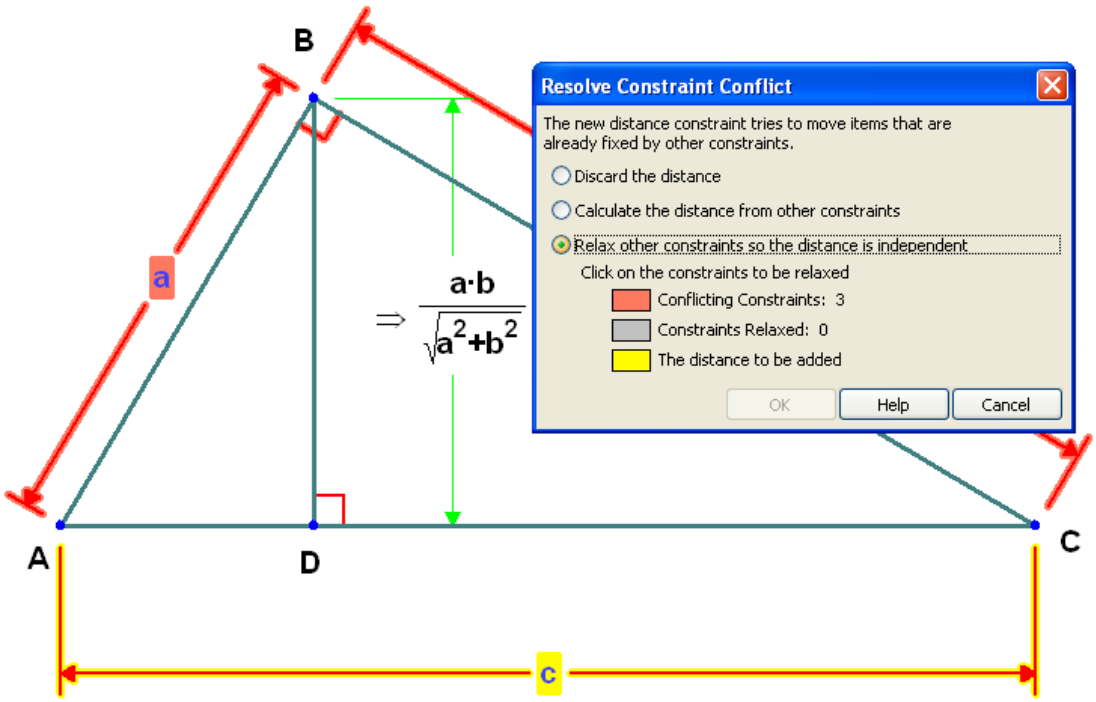
Resolve a constraint conflict.

With the lengths of two sides and the included angle already constrained, the triangle is fully defined; any new constraint you add will conflict with the existing ones. While this may be obvious, you may later encounter conflicts that are less obvious. This task takes you through the steps necessary to resolve such conflicts.

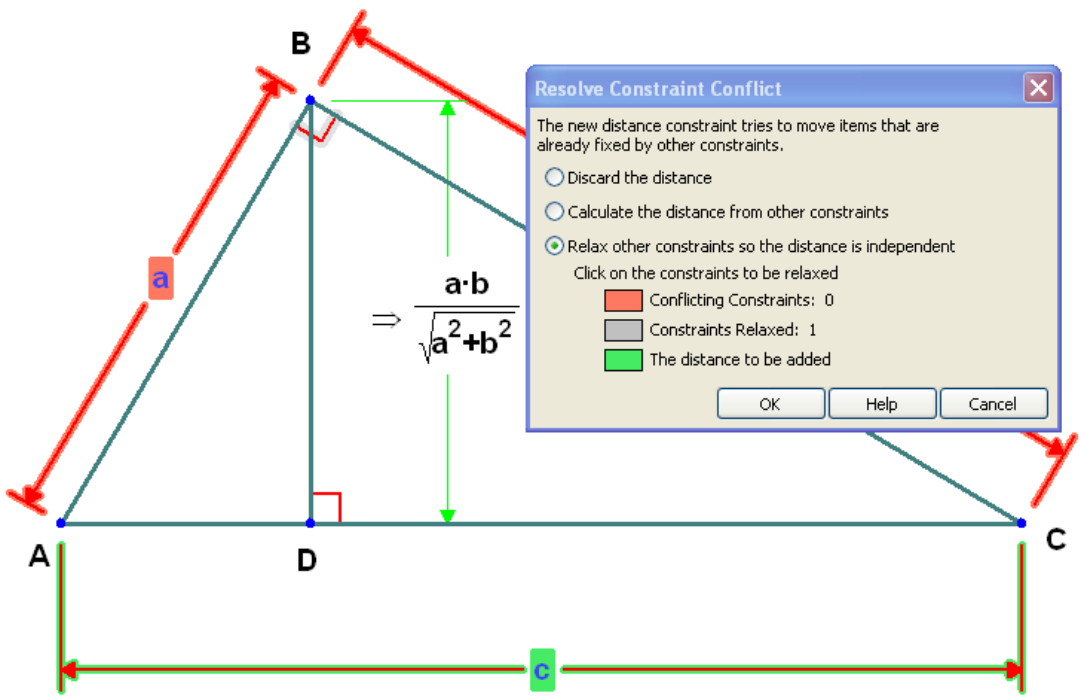
1. Select the line AC and add a length constraint. You'll see this dialog:



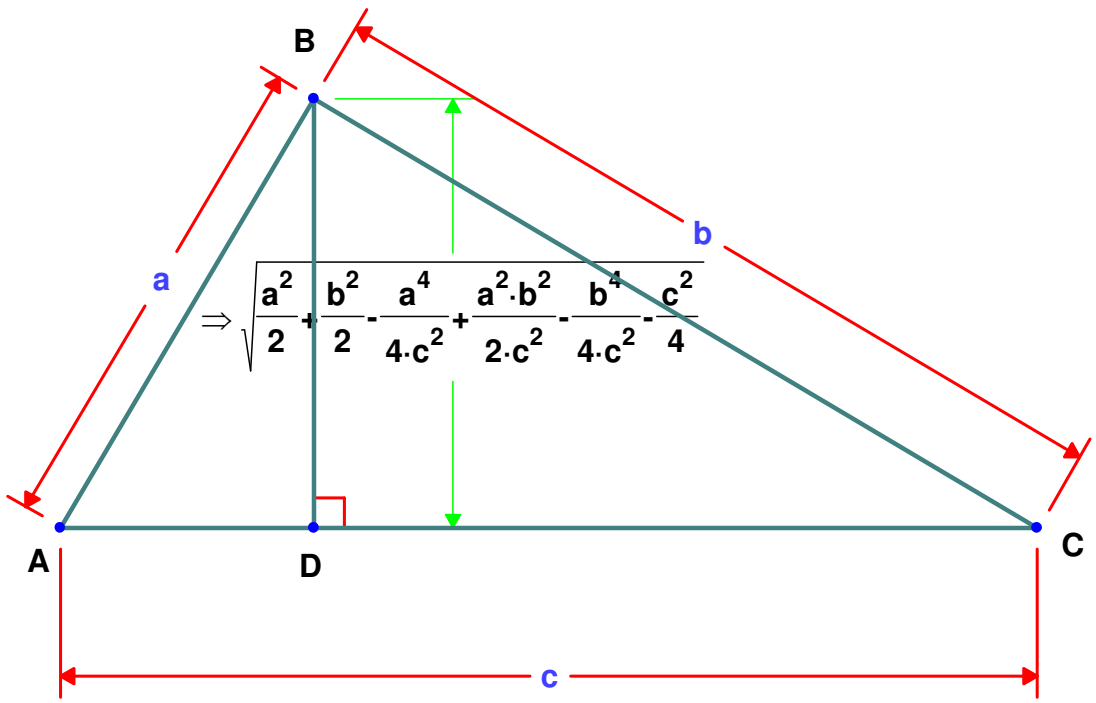
2. The dialog offers you three choices:
   - *Discard the distance*. This will close the dialog and put you back in the position you were before you attempted to constrain the length of AC..
   - *Calculate the distance from other constraints*. This will change your attempt to constrain the length to a request for the length to be calculated for you.
   - *Relax other constraints so the distance is independent*. This will give you an opportunity to remove one of the conflicting constraints so that the one you were trying to impose will become an independent constraint and no longer depend on existing constraint. You can then choose which constraint to relax.
3. Click the second radio button, *Calculate the distance from the other constraints,* and then click the **OK** button. You'll see the expression appear with the arrow (=>) that indicates it's an output. (It may also be given a name. This will be the case if the **Show Name** box is checked in the **Output** palette.
4. **Edit > Undo**
5. Try to add the new constraint again. This time, click the third radio button, *Relax other constraints so the distance is independent.* As soon as you click it, the conflicting constraints highlight in red and the new one in yellow:



6. Click on the angle at B. Its highlight changes to indicate it's selected; the conflicting constraint highlights vanish, and the new one now highlights in green.



7. Click **OK**. Now the figure includes the length constraint c, while the perpendicular angle constraint at B is gone.



8. **Edit > Undo** again. We're going to need that perpendicular angle later. Leave the drawing as it was, without length constraint *c.*

Make your own expression.

Geometry Expressions can calculate output as expressions, but you can also define and add your own expressions to a drawing.

1. Select the two points A and D.
2. Calculate the distance between them as a symbolic output.
3. Select the expression you've just output.
4. In the Output panel, check **Show Name**.

   To the left of the arrow, the name z_n appears. (Because you may have created one or more of these already while exploring, we can't predict the exact number in the subscript.) This is the symbol the application assigns by default as the name of the expression.

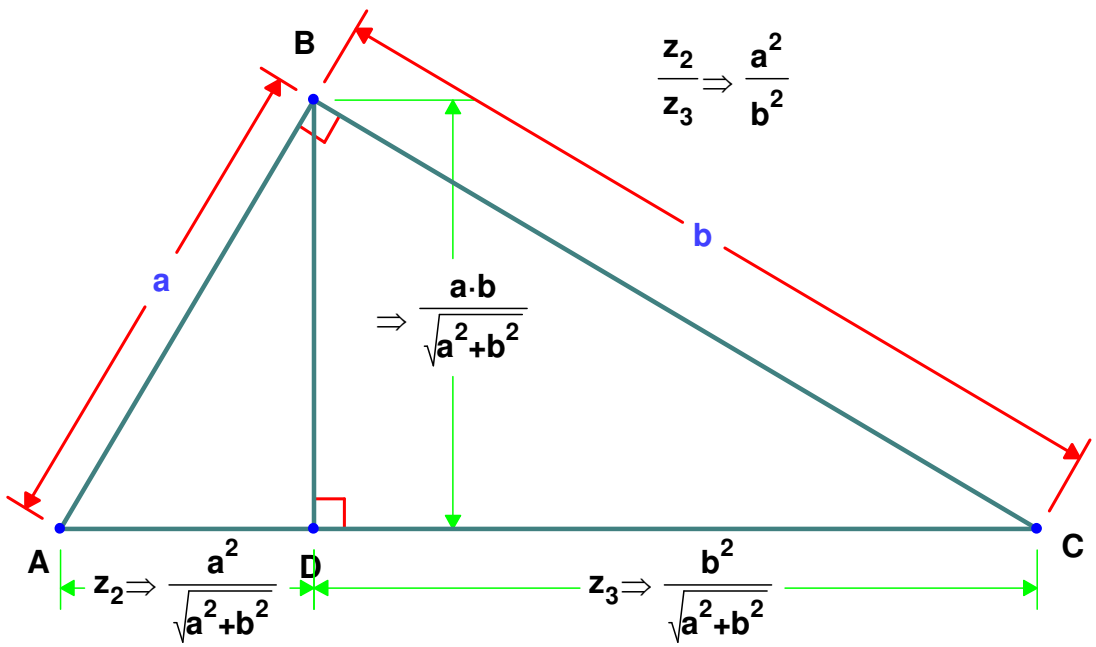
5. Select the two points D and C, and repeat the previous three steps to get a second expression, this one named z_{n+1}. (Subscripts increase by one each time the symbol they apply to is reused.)

   Using these names, you can make an expression of your own to refer to these distances. For example, you can calculate the ratio of the two lengths.

6. In the Draw palette, choose the Expression tool.
7. Click in the drawing where you want to put the expression. The expression appears as a 0, selected for overwriting. To replace the zero, enter:

   `z[n]/z[n+1]`

   (Square brackets indicate subscripts. Replace the *n* and *n+1* with the subscripts for your own named expressions.)
8. The expression is calculated and displayed.



Tutorial 2: Create and manipulate a curve.

This tutorial lets you try some of the more advanced features of Geometry Expressions. In this tutorial, you will:

— create a locus,

— lock a variable's value to see the effect it has on the drawing,

— animate the drawing by setting start and stop values for a variable,

— calculate two kinds of equations for the locus,

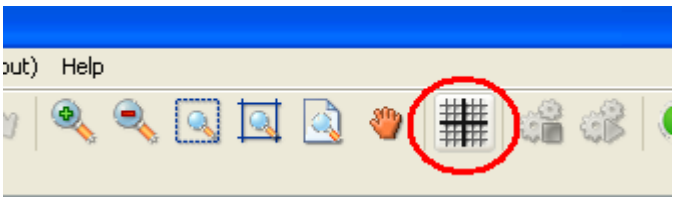
— reflect the locus in a line, and

— compare the equations for the locus and its reflection.

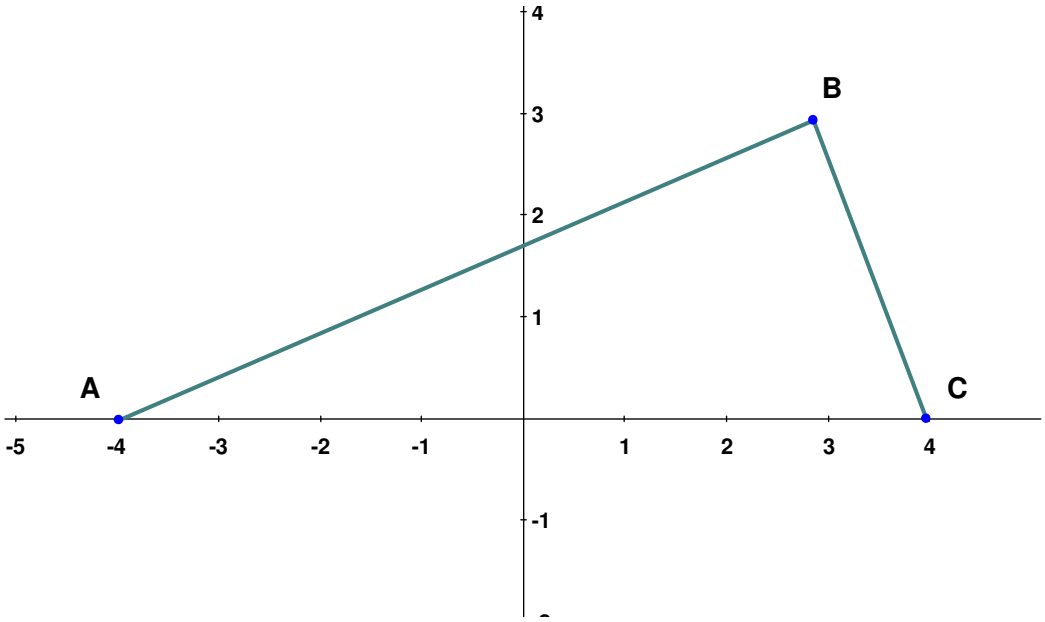
Draw a locus.

We're now going to re-create the old exercise of drawing an ellipse using a pencil, two pins, and a piece of string.

1. Turn on the axes by clicking on the grid tool in the toolbar.

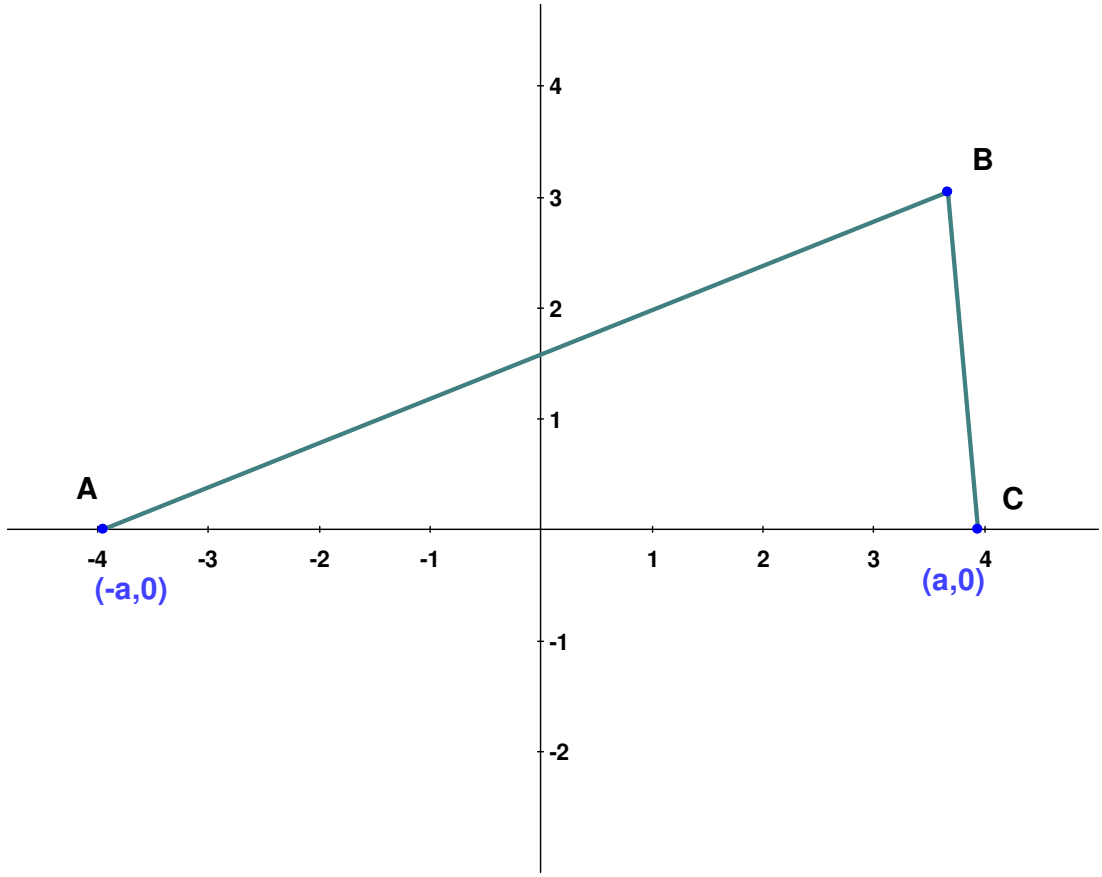


2. Make two line segments, AB and BC. Points A and C are the pins, while B represents the pencil.

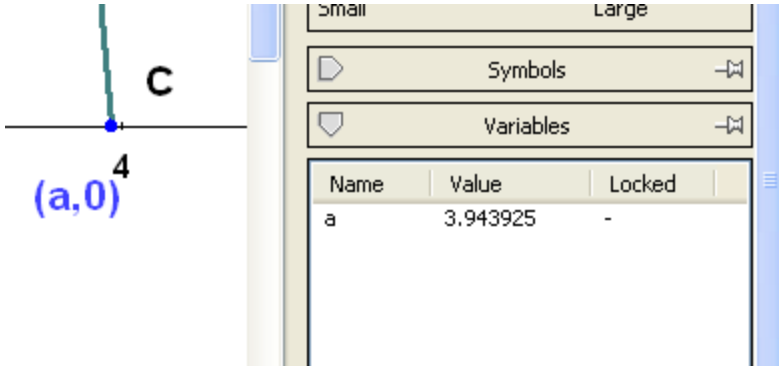


3. Select point A.

4. Add an input constraint of type *Coordinate* to set the coordinates of the point.
5. When the constraint highlights, enter (not forgetting the parentheses):
   (-a, 0)
6. Select point C and constrain its coordinates to be at (a,0).



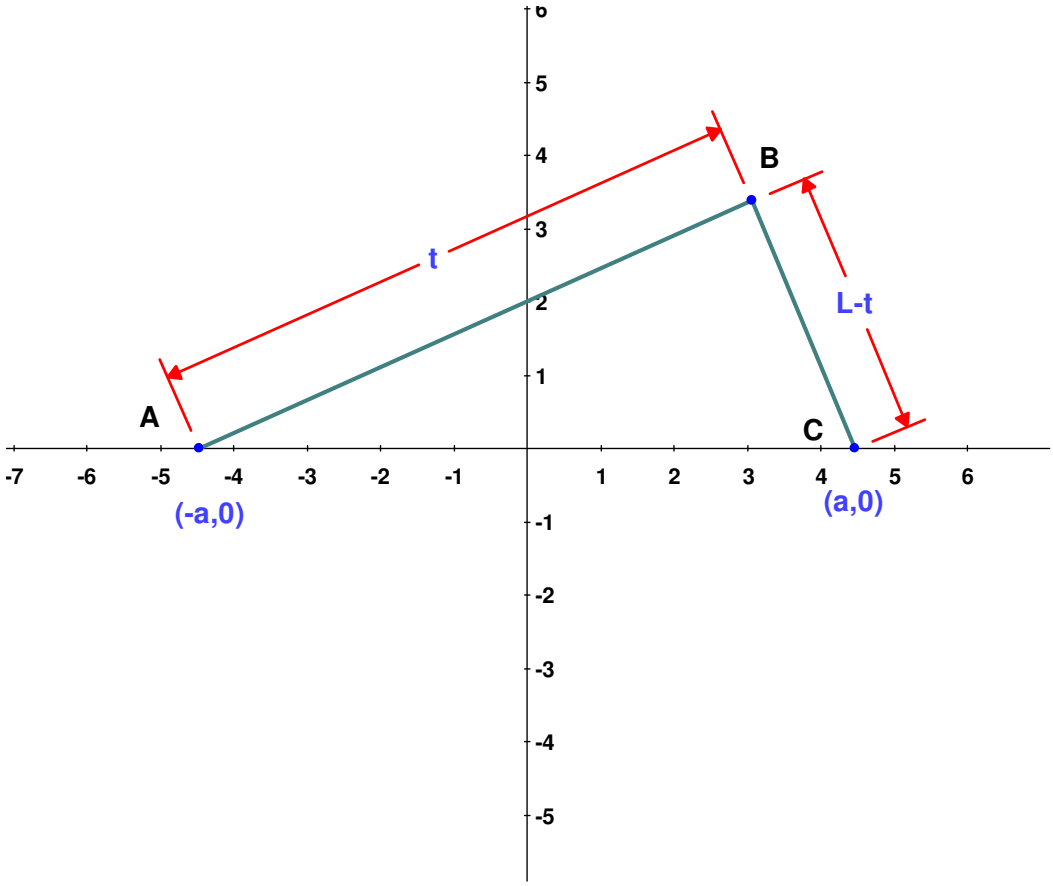
7. Scroll down, if necessary, to see the value of *a* in the Variables palette.



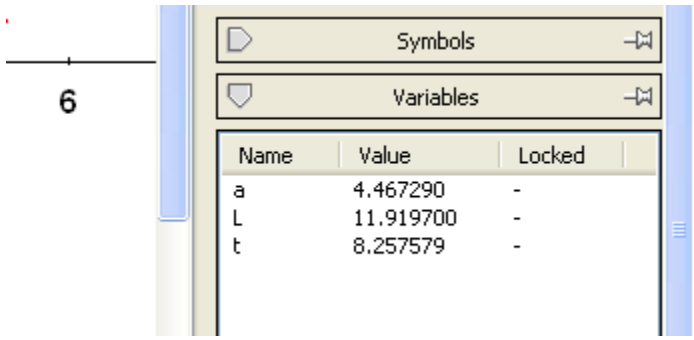
8. Reselect A and drag it to the right a short distance. As you drag, notice:
   — A is now constrained to lie along the X axis; moving it up or down has no effect.
   — When you move A, C also moves.

— When you move A, the value of the associated variable, *a, changes* in the Variables list.

9. Select the line AB and constrain its length to be distance *t*.
10. Select the line BC and constrain its length to be *L–t*. *L* now represents the length of the string.

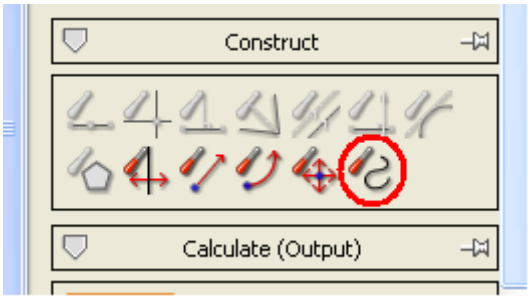


11. Notice that the two new variables have appeared in the Variables list.



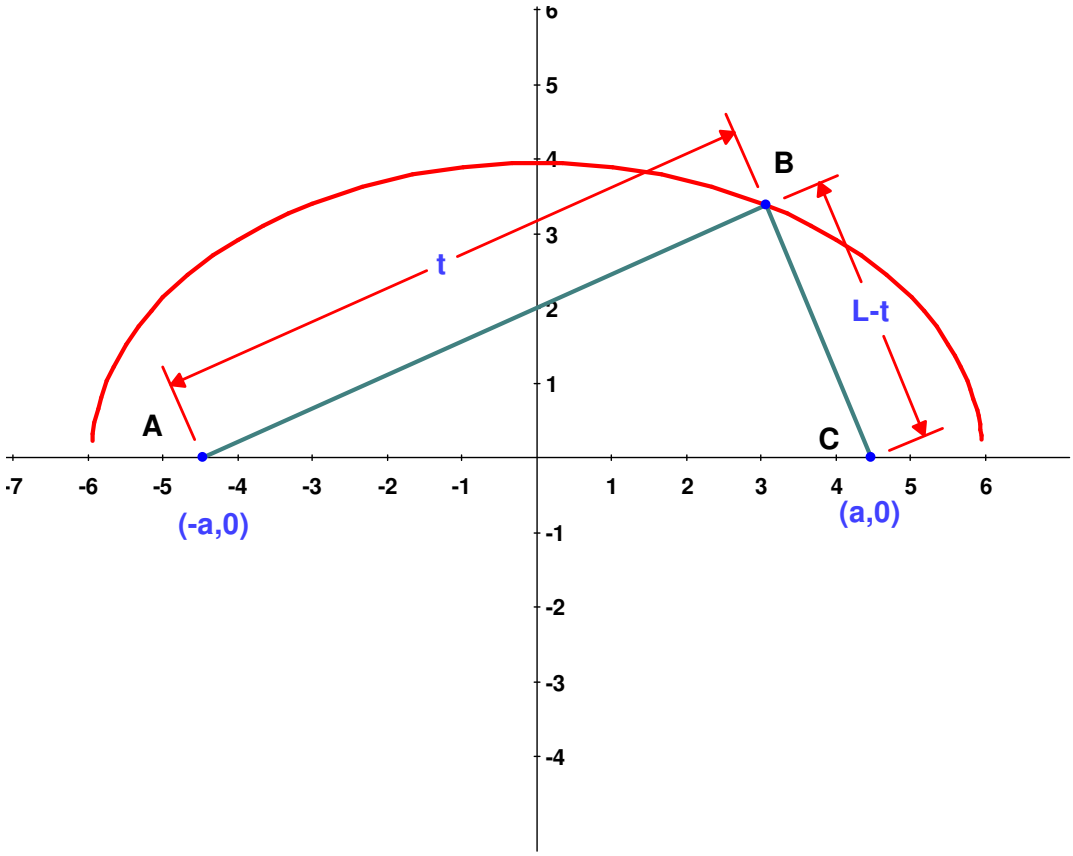
12. Select B.

13. In the Construct palette, select Locus to construct a locus through B.



14. In the resulting dialog, choose *t* as the parameter to vary, and enter start and end values of 0–25, guesses that will probably produce a complete curve.

    The locus appears — half of an ellipse, above the X-axis.

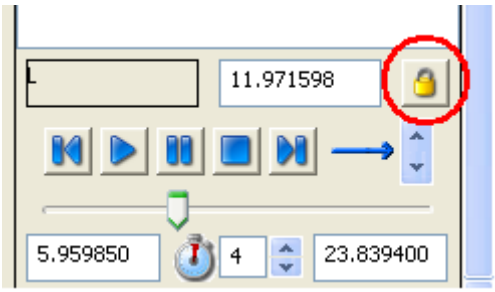


15. Now drag B up and down, the equivalent of changing the length of the string, and notice how the ellipse changes. You can also drag A or C to change the position of the pins.

Lock a variable.

When you draw an ellipse this way using real pins, pencil, and string, the length of the string can't change. To emulate this real-world behavior, we can lock the value of the variable *L*.

1. In the Variables list, select *L* and click the lock icon below the list. A plus sign (+) appears next to the locked variable in the Locked column.

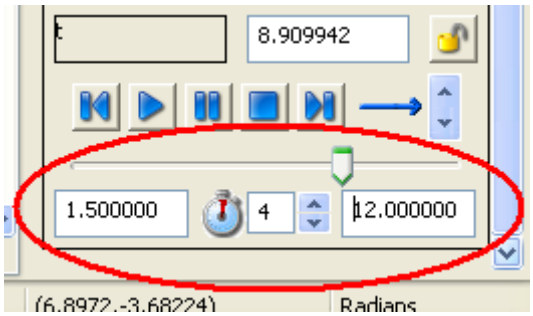


2. Select points A or C and drag it. With the length of the string constant, the locus will change accordingly.

Animate a variable.

You can also animate the drawing. To do so, we can set start and stop values for the variable *t.*

1. In the Variables list, select *t*.
2. The first input field below the video playback interface specifies the start value. Enter 1.5.
3. The middle field specifies the number of seconds for one cycle of the animation. Accept the default value of 4. The input field to its right specifies the stop value. Enter 12.



4. To animate the drawing, click the **Play** button ( ).

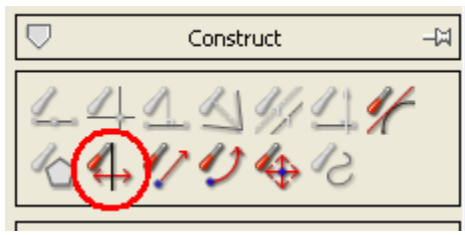
NOTE: The start and stop values that yield an interesting animation depend to some extent on where you've located objects in your drawing. Experiment to find values that make an interesting animation. Values that do not allow GE to create a construction sequence will cause objects to disappear briefly. They'll reappear when values make sense again, or when the animation stops.

Calculate parametric and implicit equations.

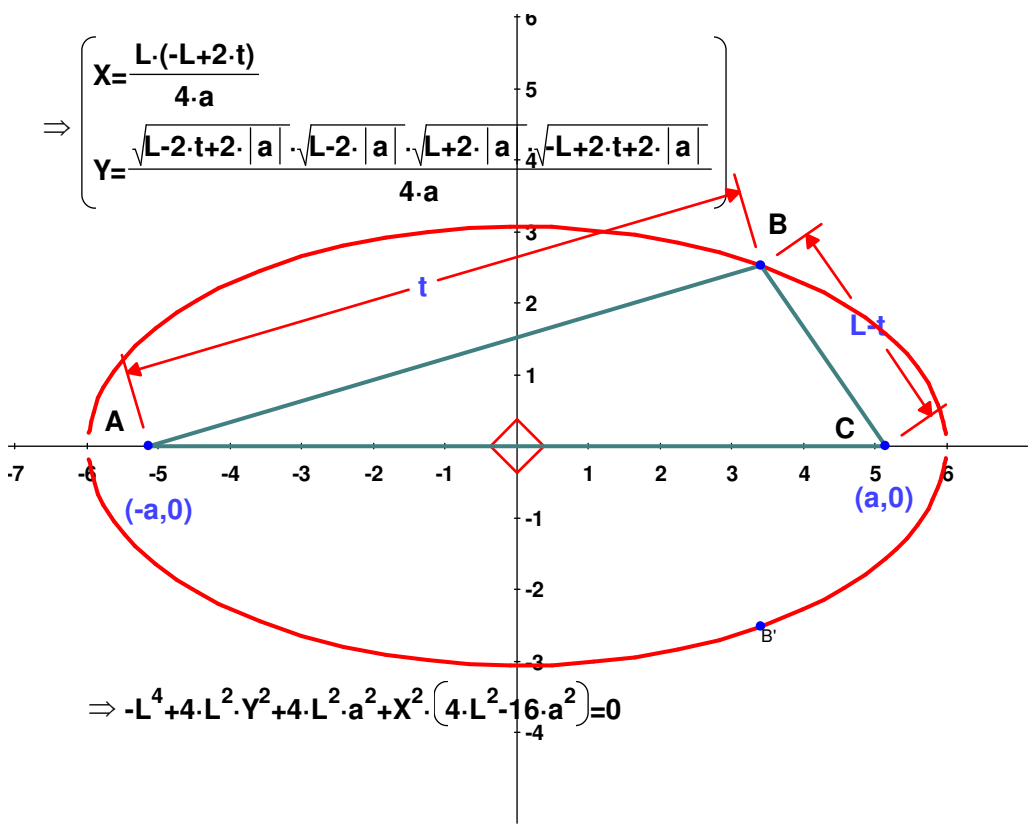
1. Select the locus. In the Calculate (Output) palette, chose Parametric equation. The resulting expressions are the formulas for *x* and *y* as a function of *t.* Move the expression wherever you wish on the drawing.
2. Select the locus again and, this time, request the Implicit equation. The resulting expression gives the equation of the curve in terms of *x* and *y.*

Make the locus reflection.

1. Select the line segment tool again and make a line from A to C. This line lies on the X-axis.
2. Select the curve.
3. In the Construct palette, choose Reflection. The status bar now shows a message prompting you to choose the axis about which to reflect the curve.



4. Select the line segment AC. This will reflect the curve about the X-axis.



B is also reflected, its reflection appearing on the lower curve as B'.

5. Select B and drag it; B' follows. But, though you can select B', you can't drag it; it's locked as the reflection of B.

Compare locus and reflection equations.

1. Select the reflected curve and request its parametric equation (from the Calculate (Output) palette).
2. Compare them side-by-side and you'll see that the x-parameterizations are the same and the y-parameterizations differ only in sign (which is, of course, as it should be).
3. If you repeat the experiment for the implicit equation, you'll see they are identical.

Explore on your own.

Having completed the introductory tutorials, we hope you'll be comfortable exploring on your own. If you wish, further interesting examples can be found on our website, www.geometryexpressions.com.

III. Examples

We present a few worked examples using Geometry Expressions. Corresponding example files are installed with Geometry Expressions in the Examples folder.

Many more examples are available at the web site:

www.GeometryExpressions.com

Example 1: Circumcircle Radius

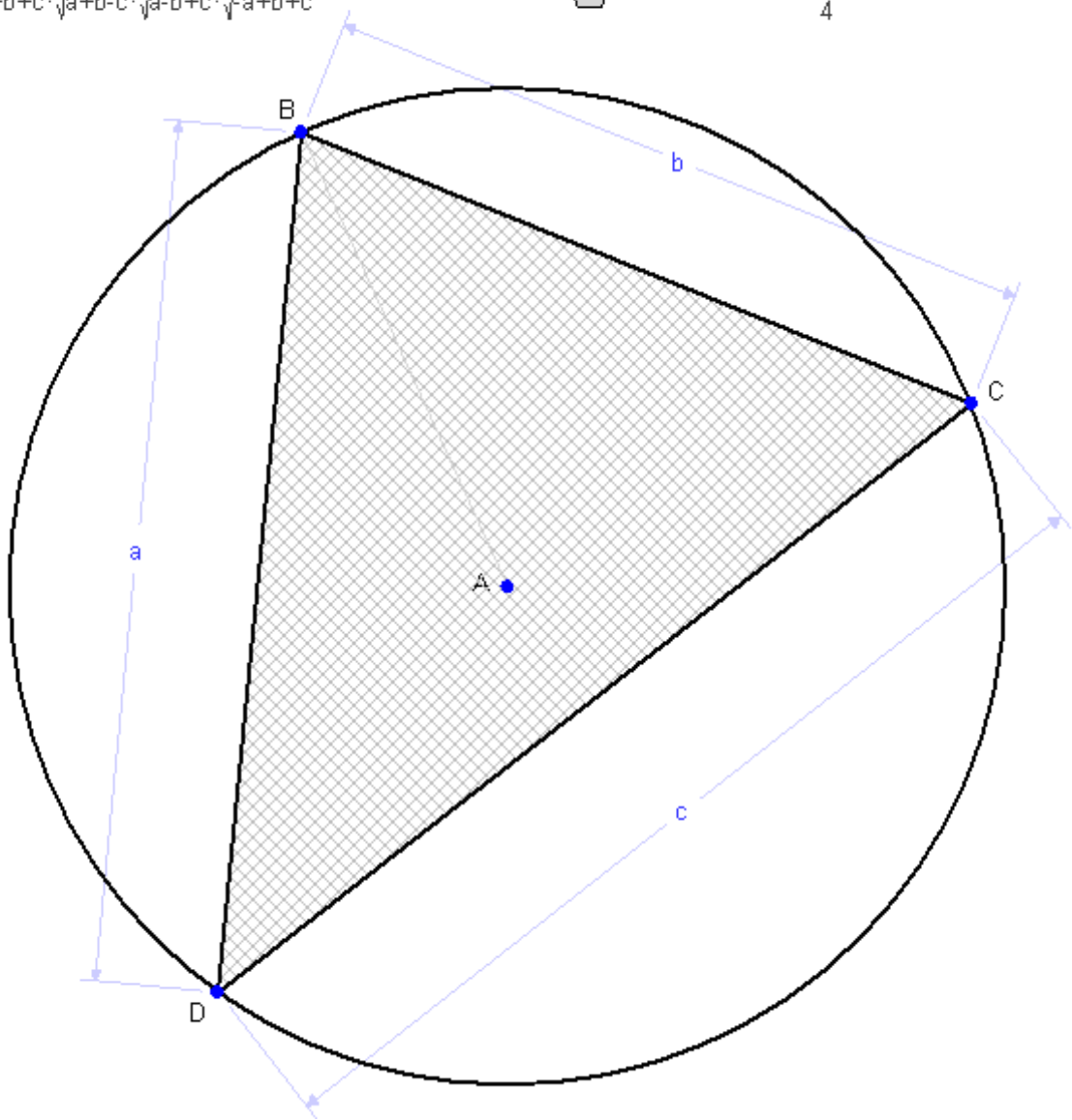
Measure the area of a triangle with sides length a,b,c, and measure the radius of the circumcircle.

circumcircle radius

$$\Rightarrow \frac{a \cdot b \cdot c}{\sqrt{a+b+c} \cdot \sqrt{a+b-c} \cdot \sqrt{a-b+c} \cdot \sqrt{-a+b+c}}$$

triangle area

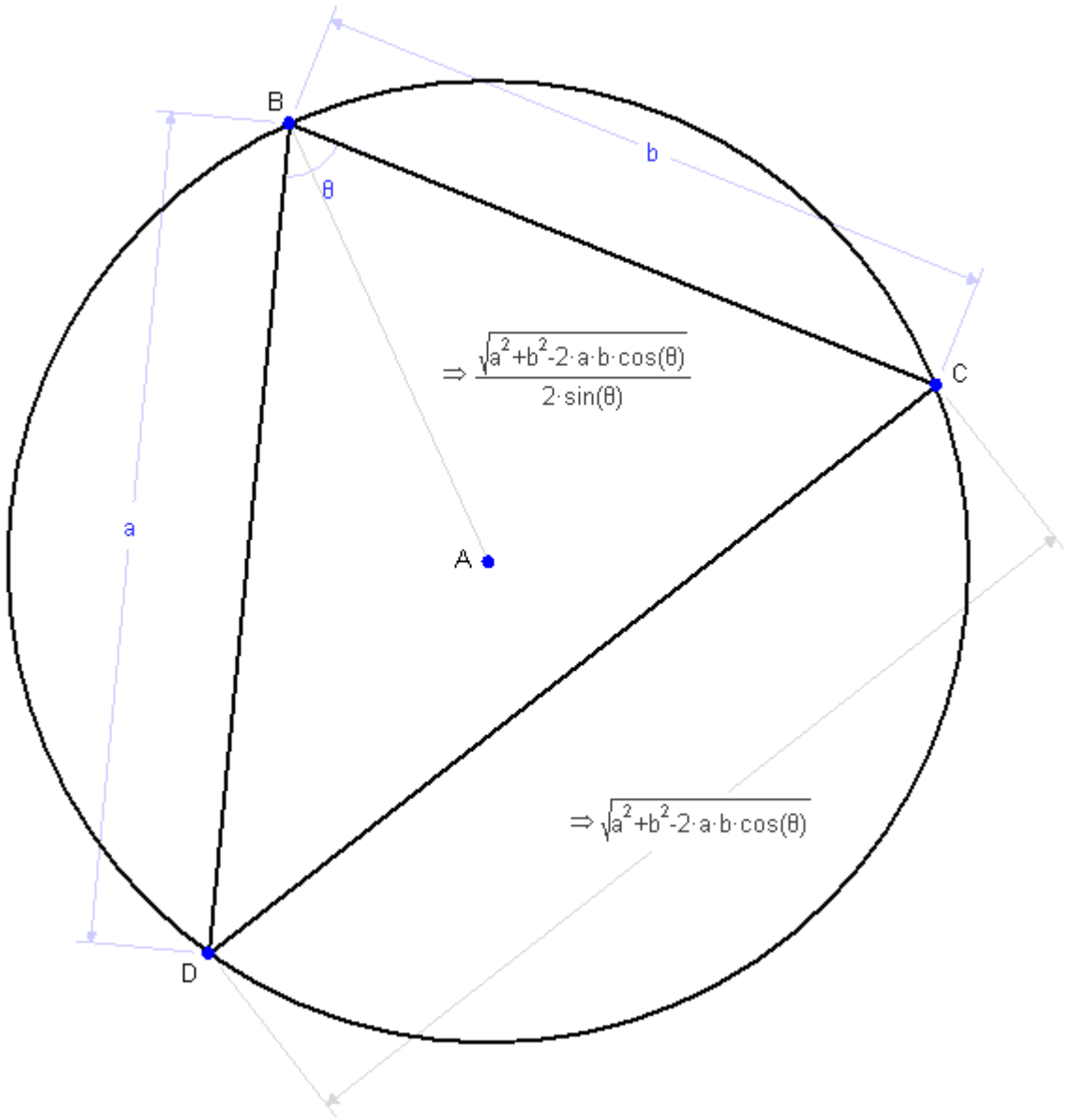
$$\Rightarrow \frac{\sqrt{a+b+c} \cdot \sqrt{a+b-c} \cdot \sqrt{a-b+c} \cdot \sqrt{-a+b+c}}{4}$$



What is their relationship?

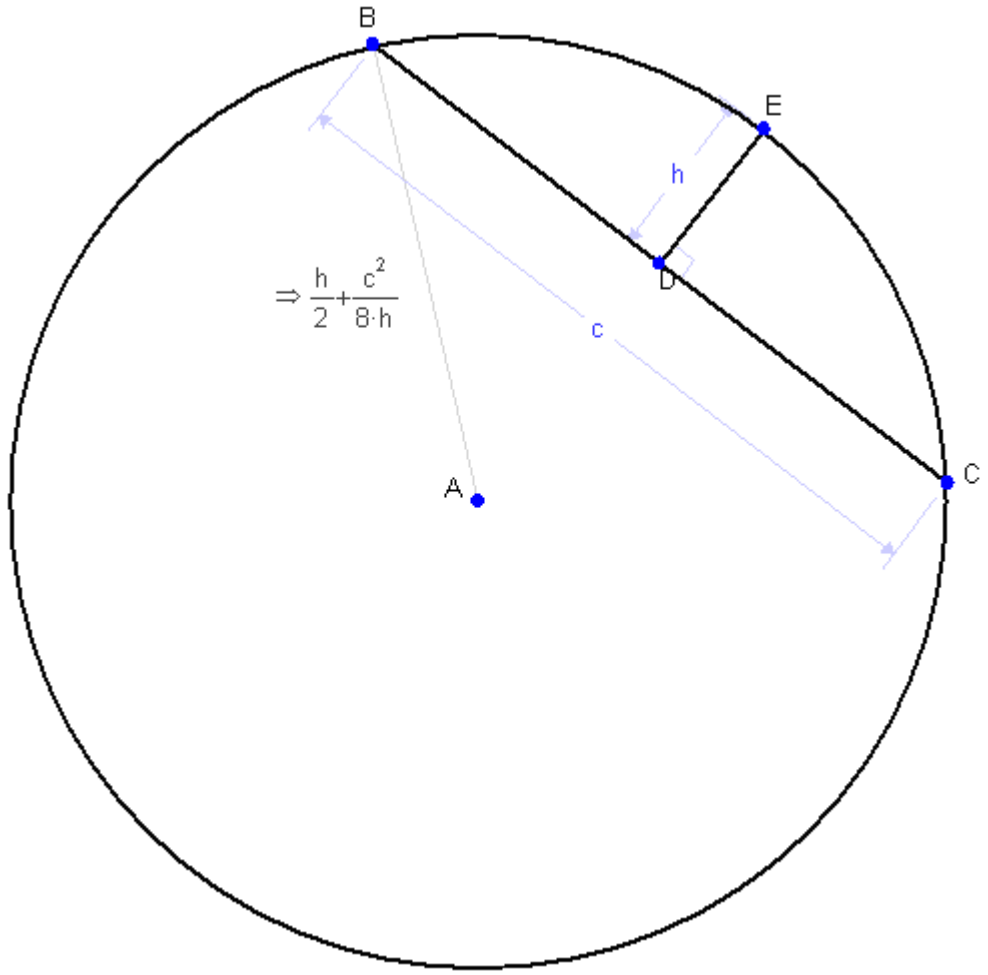
If a triangle is defined in terms of two sides and the included angle, what is its area?

Hence you can derive a formula for the radius of the circumcircle which involves an angle:



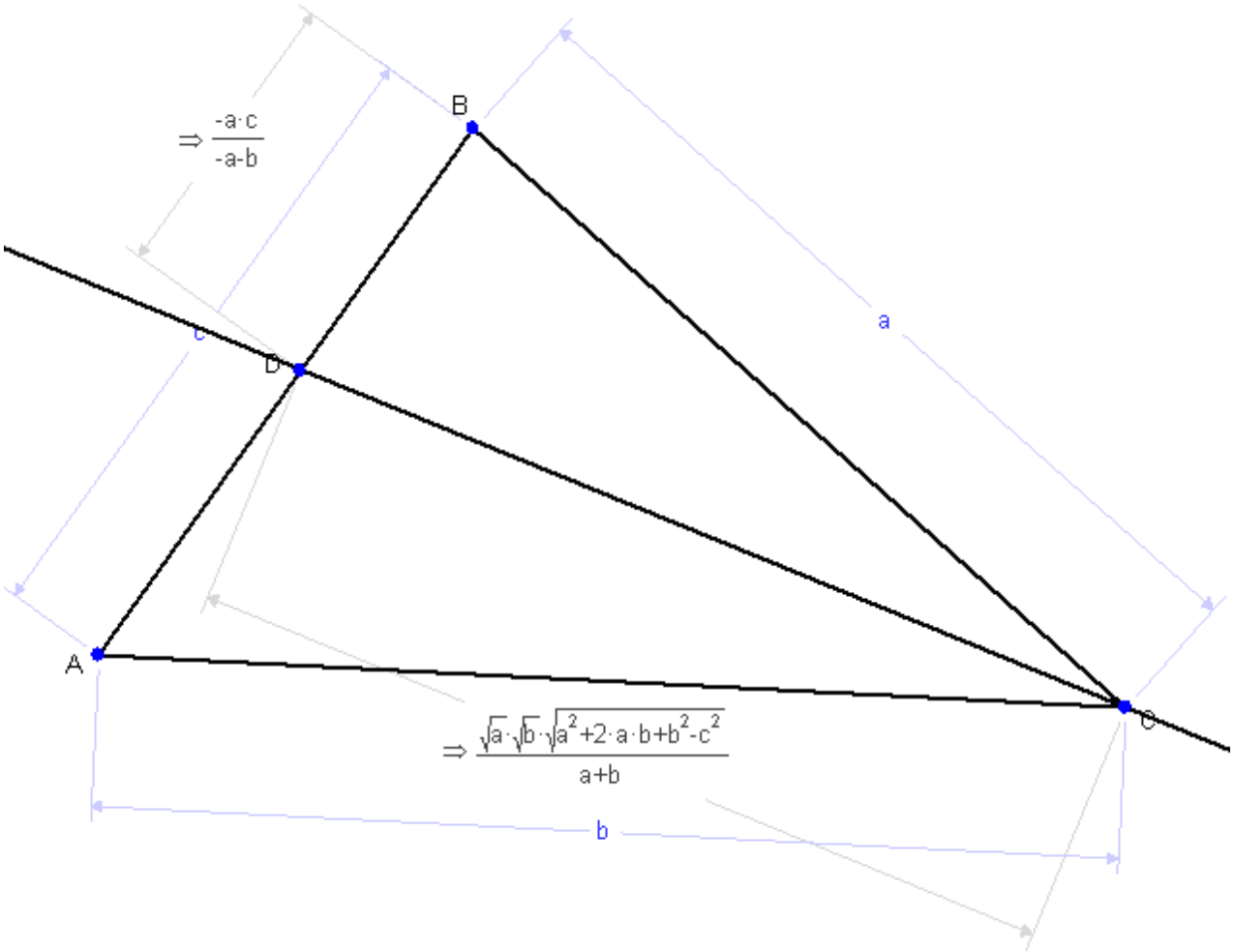
Example 2: Radius of arc offset from line segment

Imagine a piece of drawing software where the user creates an arc by dragging the center of a line segment perpendicular to the segment. The software author might want a formula for the radius of the arc given the length of the segment and the distance the user has dragged the center away from the segment.



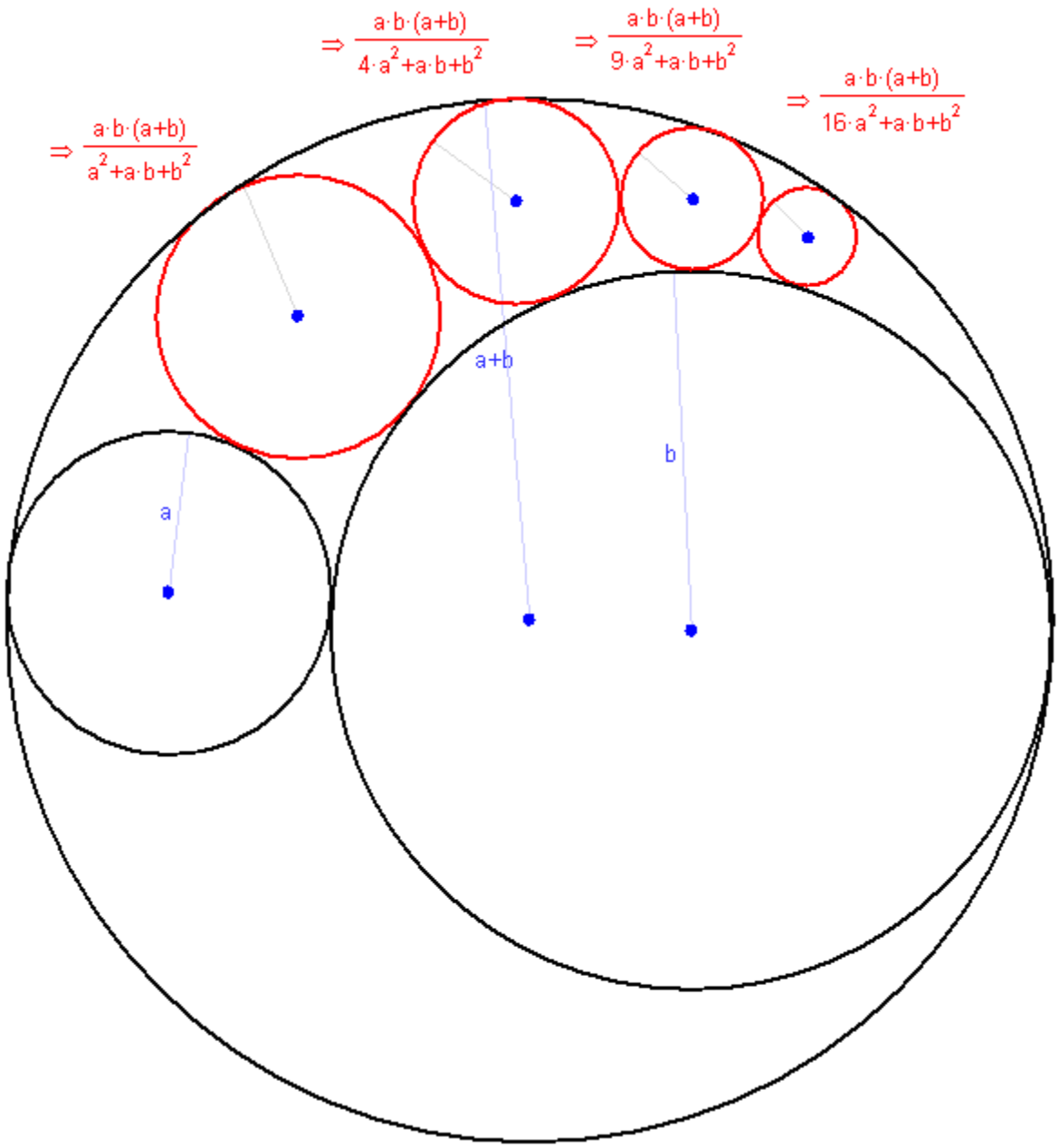
Example 3: Length of the Angle Bisector of a triangle

Given a triangle with side lengths a,b,c, what is the length of a perpendicular bisector?



Example 4: A Family of Circles

We look at the family generated by two circles of radius a and b inside a circle of radius a+b:



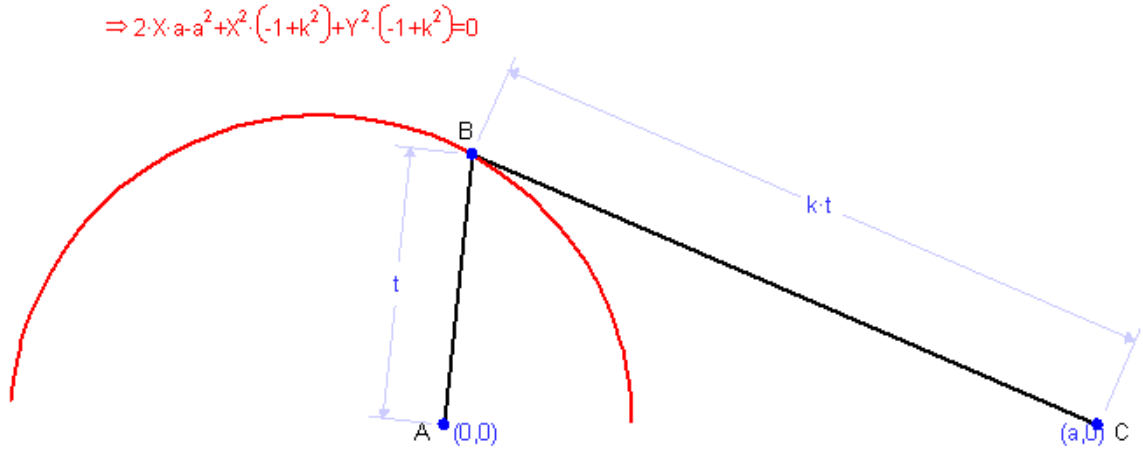
Clearly the radius of the nth circle is:

$$\frac{ab(a+b)}{n^2a^2+ab+b^2}$$

Can you prove this?

Example 5: Circle of Apollonius

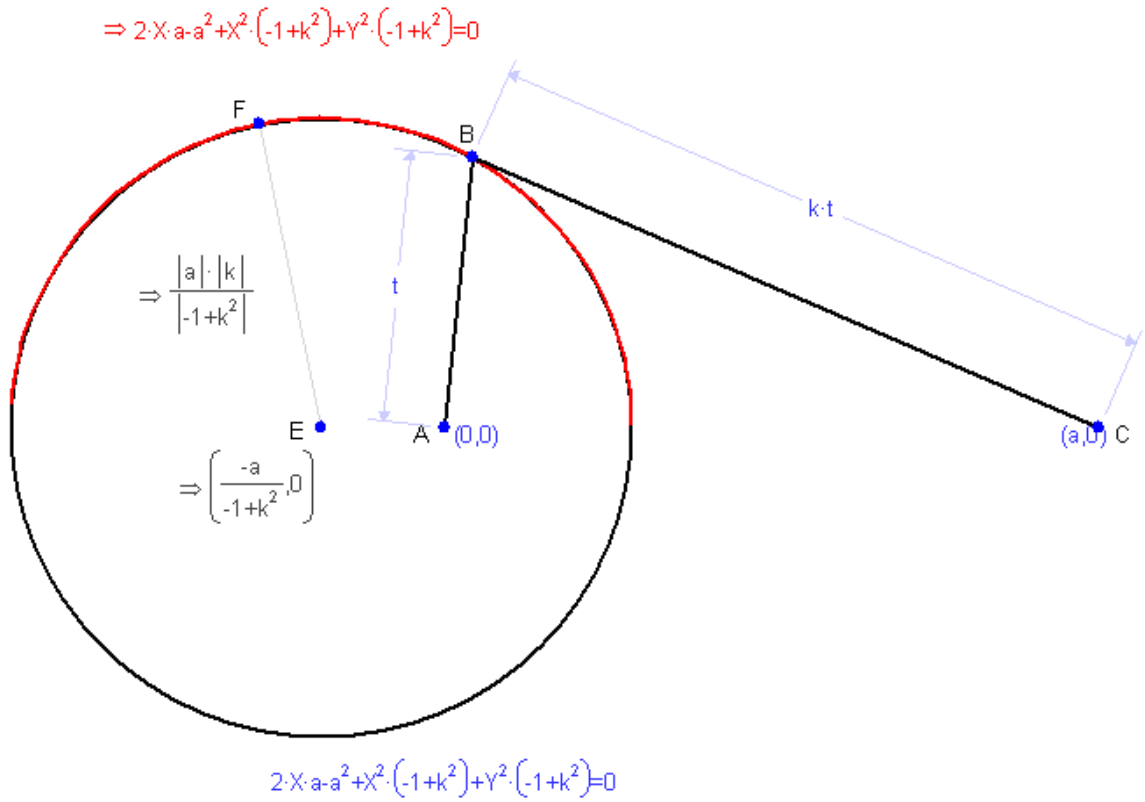
The Circle of Apollonius is the locus of points the ratio of whose distance from a pair of fixed points is constant:



How do we know this is a circle?

What is the center and radius?

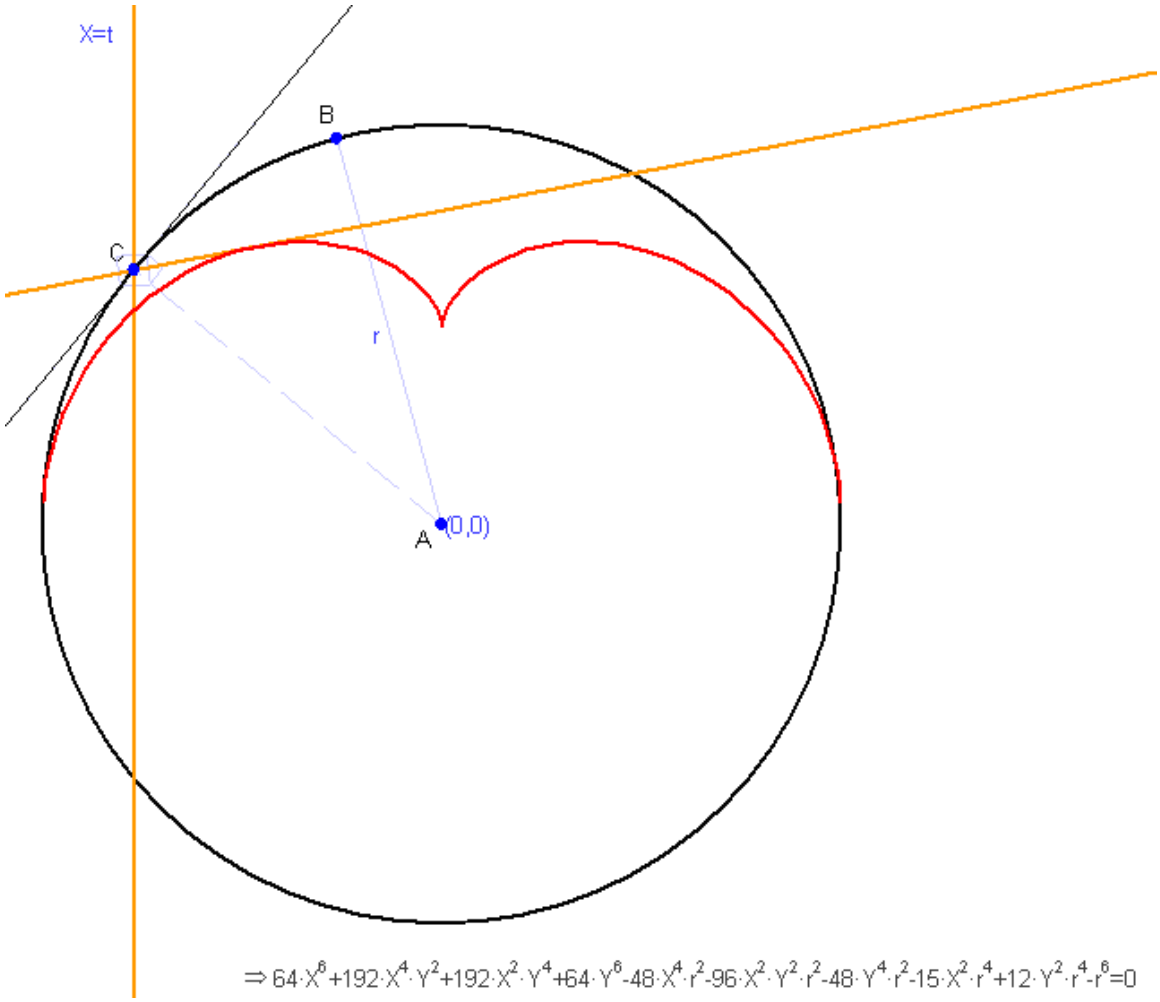
You can always get Geometry Expressions to tell you: draw a circle and set its equation to be the same as the locus equation (copy and paste works fine). You can then read off the center and radius.



Example 6: Caustics in a cup of coffee

The Nephroid curve generated by reflecting a set of parallel rays in a circle, and then taking the envelope of the reflected rays.

(An envelope is created in Geometry Expressions as the locus of a line or line segment).

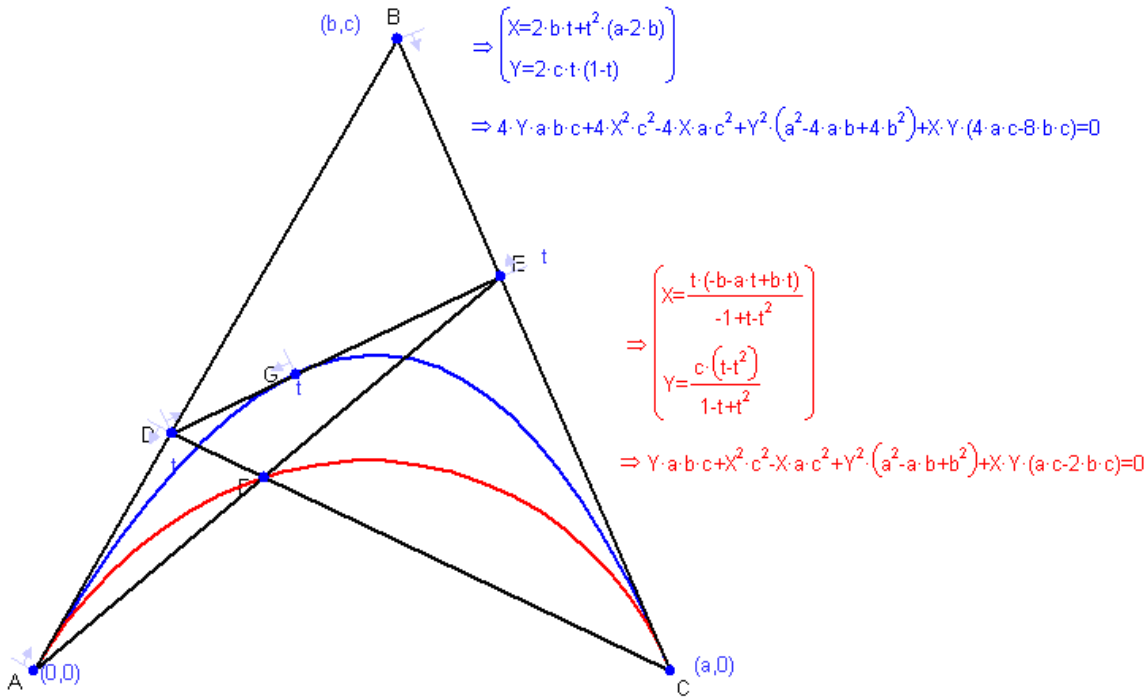


This curve is the caustic which you'd observe if you shone a light obliquely into a cup of coffee.

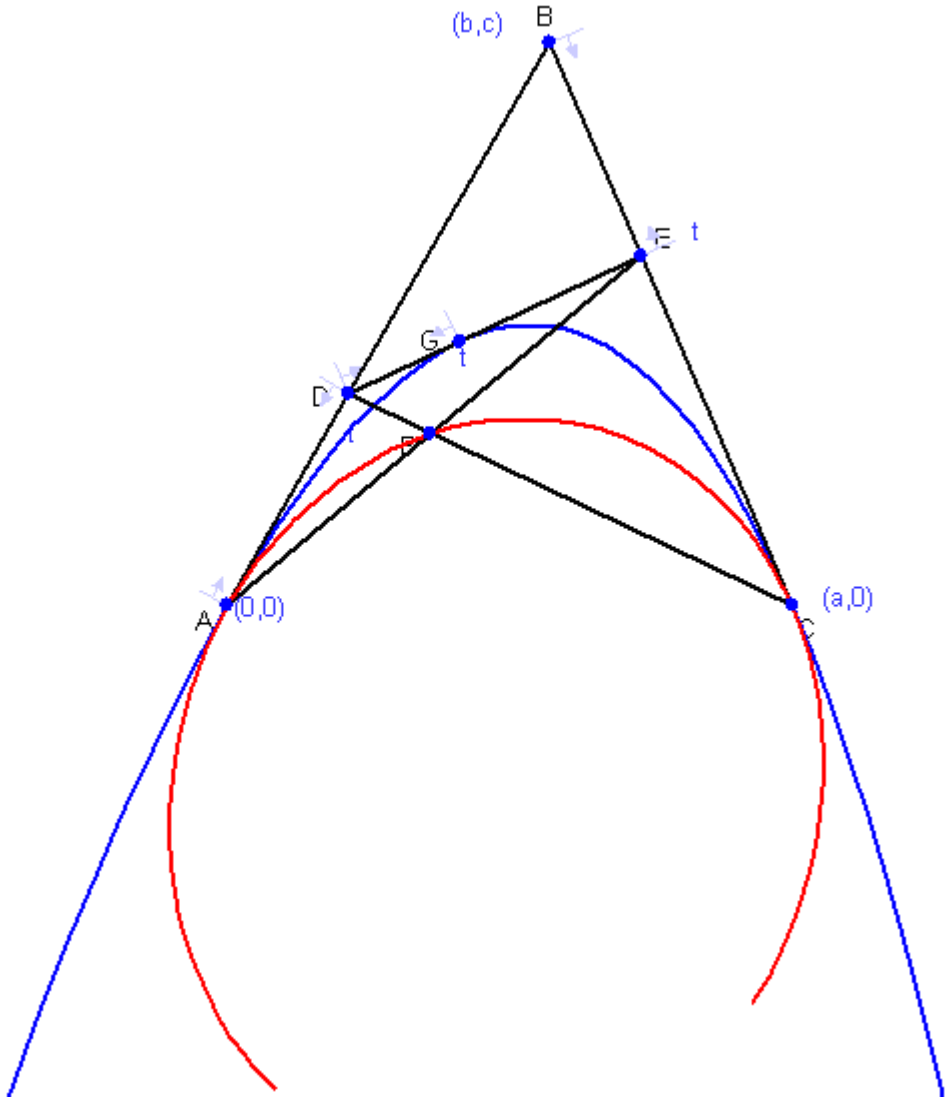
Example 7: Two Quadratic Splines

In the diagram below, D and E are located at proportion t along AB and BC respectively. F is the intersection of AE and CD. G is the point proportion t along the line DE. We examine the loci of F and G as t varies from 0 to 1.

Observing the parametric form of the curves we see that one is a parametric quadratic, while the other is a rational quadratic. Implicit forms are both conics (and almost, but not quite, identical).



What types of conics are they? Extending the curves a little can give a clue:



The upper curve looks like a parabola, the lower certainly does not.

Can you show this is generally true from the algebraic equations of the curves?

IV. User Interface Reference

Need Help?

There are many ways to get help with Geometry Expressions.

Your installation comes with *Geometry Expressions* Manual, in PDF format suitable for creating a hard copy. The manual contains these sections:

- Solving Geometry Problems with *Geometry Expressions* – a detailed explanation of the basic concepts.
- *Geometry Expressions* Tutorials - step by step instructions.
- Examples - several examples chosen to illustrate the power and features of the software.
- Reference - a copy of the embedded Help system.

In both the printed manual and the embedded Help system you can:

- Use the table of contents to get details on using a tool, an icon or a menu.
- Use the index for help on a particular topic, such as "parametric equations".

Inside the Geometry Expressions Help system you can:

- Use the search tool to find all topics based on a key word, such as "constraints".
- Browse through help with the up "Previous page" and down "Next Page" arrows  . This will step you through the help sub-topics in a logical sequence.
- You can click on colored text to get more information.

Tool Tips:

When you move the cursor over any icon on the screen, the name of the icon appears briefly below the cursor.

The Display and How It's Organized

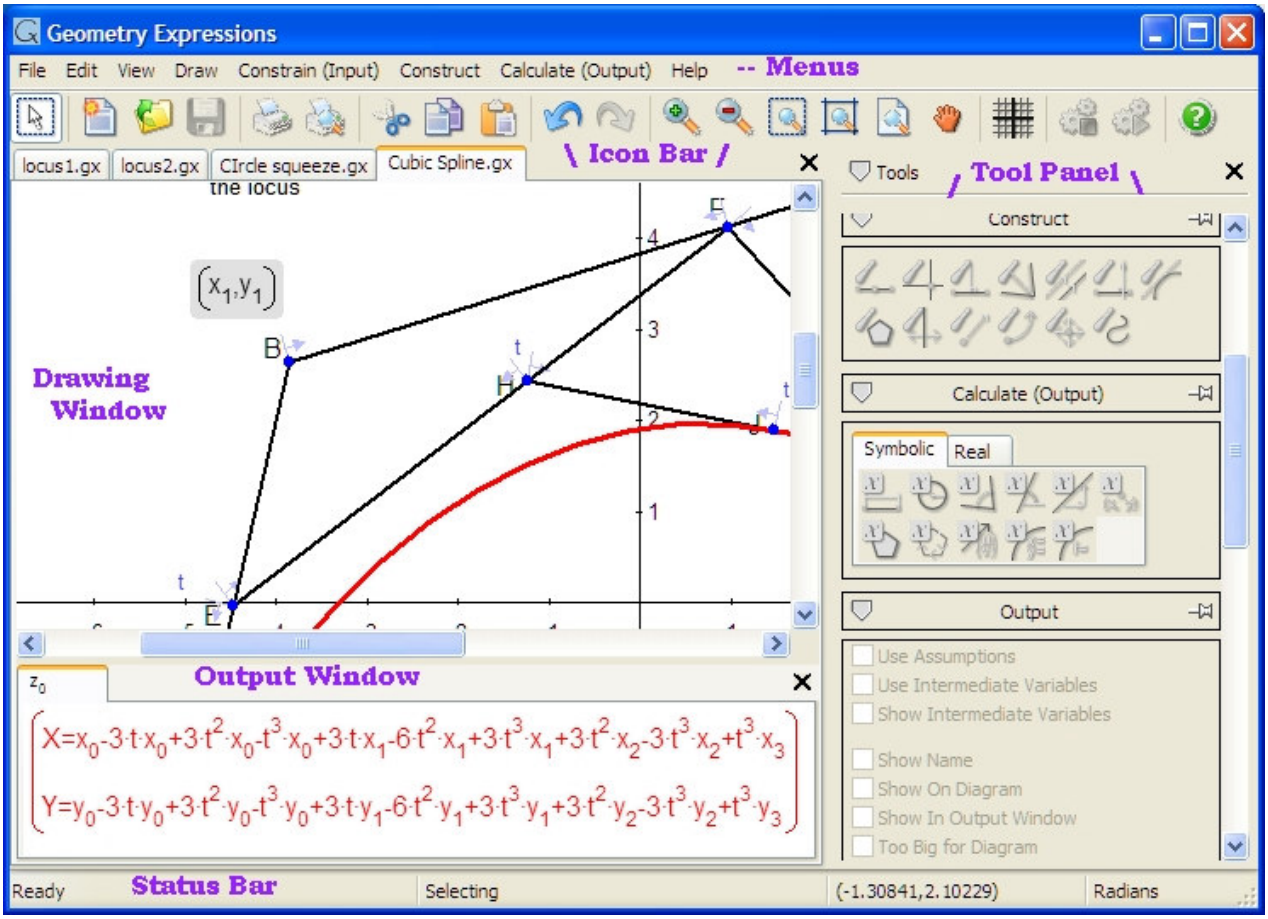
Many of the menu items in the drop-down menus at the top of the screen correspond to one of the icons or buttons across the top or down the side of the display window.

Icons across the top of the screen comprise the standard Windows File, Edit, View and Help commands. The construction and calculation tools are displayed along the side of the drawing window. These toolboxes can be displayed on the left or right side of the window, or hidden.

The **Output** window can be displayed below the drawing window to show large expressions.

The status bar at the bottom of the window displays the following (from left to right):

<Menu Help> <Current Mode> <Cursor Coordinates> <Angle Mode>



The Status Bar

The status bar at the bottom of the screen prompts the following information (from left to right):

- <u>Menu Help</u> – summary of a selected menu item.

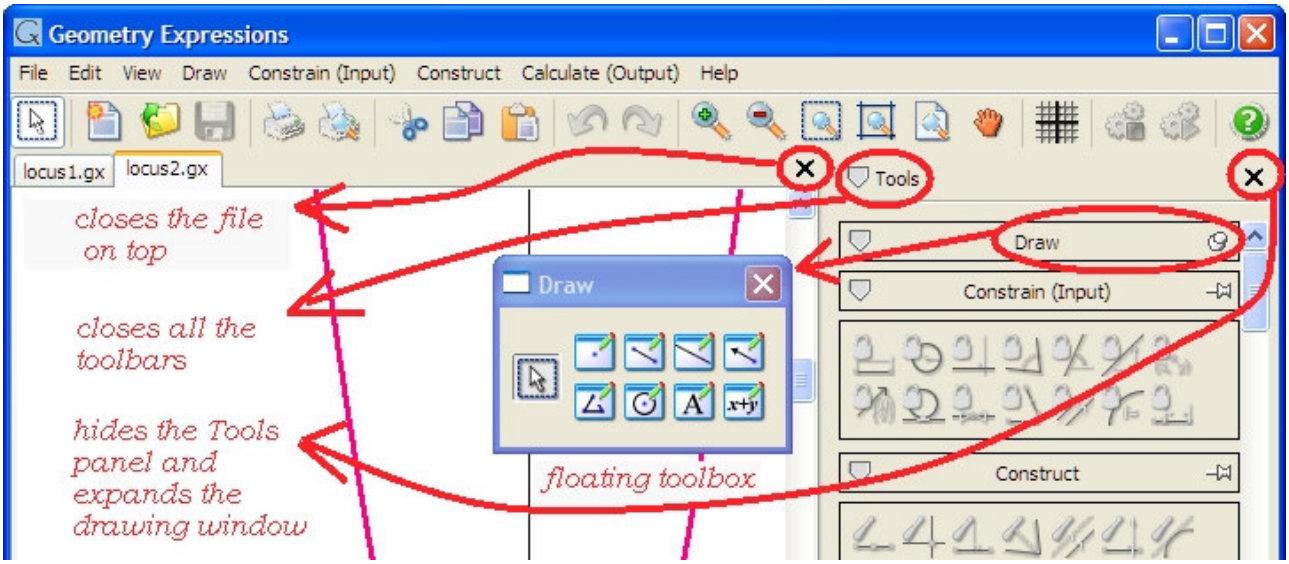
- Current Mode – Each icon in the drawing toolbox represents a mode. Tools requiring additional inputs after clicking the tool will display further prompts in this field.
- Cursor Coordinates – Displays the current coordinates of the cursor in the diagram.
- Angle Mode – Displays the current angle units used (degrees or radians). This can be set in the **Edit** / **Settings** / **Math** window.

Customizing Your Display

You can arrange the display as it suits you.

- Use **View** / **Tools** to place the **Tools** panel on the Left or Right side of the window.
- Use the arrow on the upper left side of individual toolboxes to show or hide them.
- Click the pushpin on the upper right side of individual toolboxes to make it a "floating" box that you can drag anyplace on the screen.

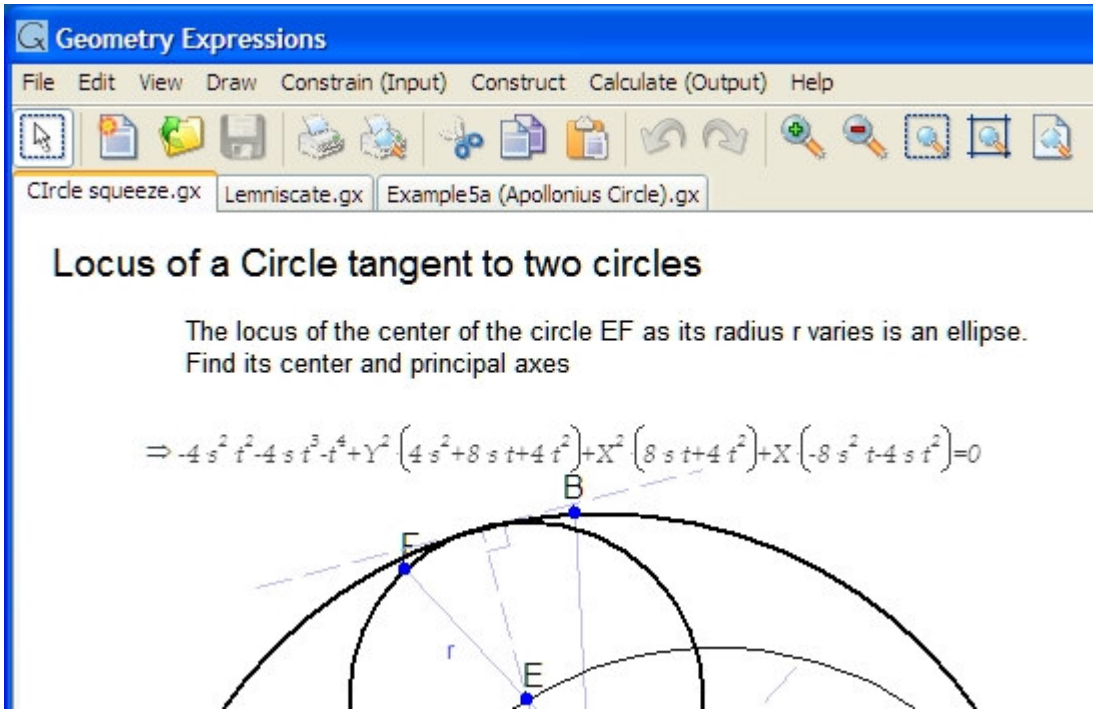
In the image below you can see other buttons which control the display.



File Handling

Geometry Expressions uses standard Windows file Open and Save operations. Save your files regularly with the handy icon at the top of the screen.

The data files generated from your drawings will have the extension ".gx". You may create multiple data files and have them open in a session. Each file is on separate page with the tabs across the top of the drawing window. Click the tab to view the file.



Working with Algebra Systems

All mathematical expressions in Geometry Expressions are written in MathML, so you can copy and paste directly to or from any other MathML program.

Usually the standard **Edit** > **Copy** or control-C command should work. If this doesn't give you the desired result, try the **Edit** > **Copy As** command. Select from the submenu of choices appropriate to your application.

In this section, we describe how to copy mathematics from Geometry Expressions into an algebra system, and from an algebra system into Geometry Expressions.

We describe how to do this generically using MathML. We also describe specifically how to share information with Maple and Mathematica.

MathML

There are 3 different ways to export expressions as MathML from Geometry Expressions:

In each method, you must first select the measurement to be output. Then either:

Copy copies the expression as both content and presentation MathML

Copy As > Content MathML copies the expression as Content MathML only.

Copy As > Presentation MathML copies the expression as Presentation MathML only.

Depending on the destination application one of these methods may work better than the others.

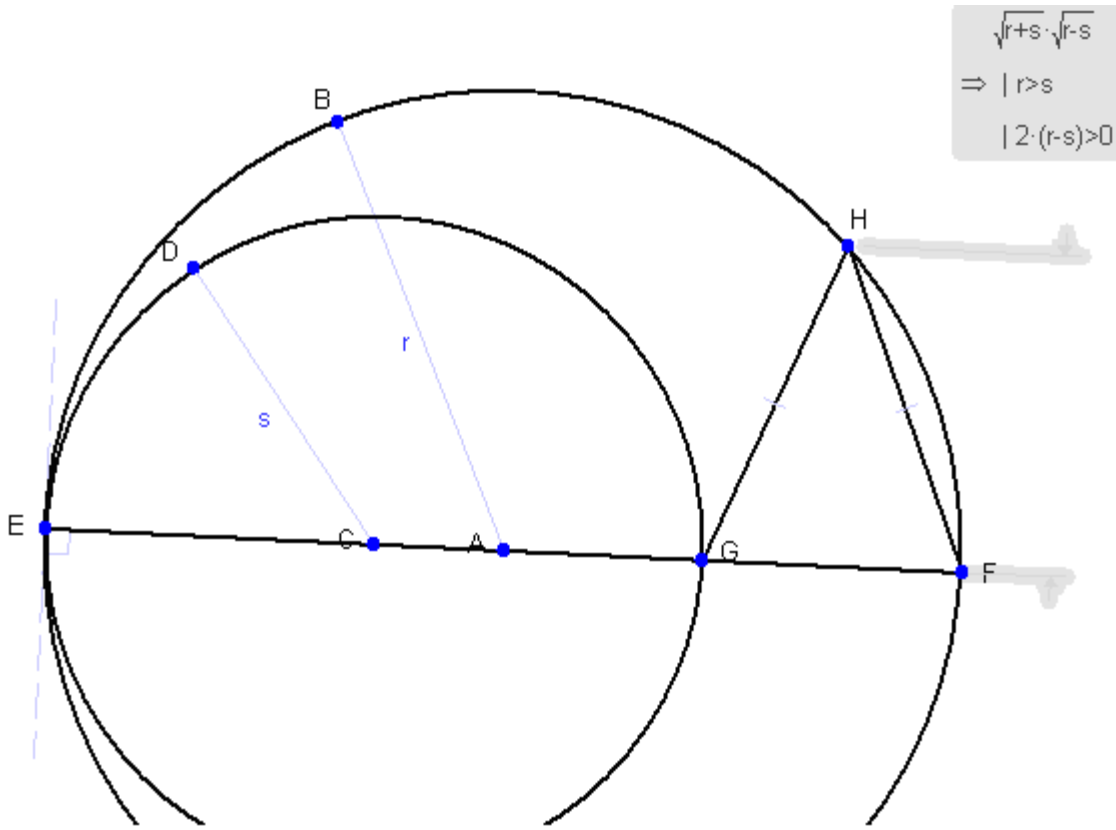
On import, Geometry Expressions is looking for expressions in Content MathML form.

Special forms of expression

There are a couple of special cases to note:

- An expression with assumptions is turned into a list, with the main expression as the head of the list, and the assumptions following.

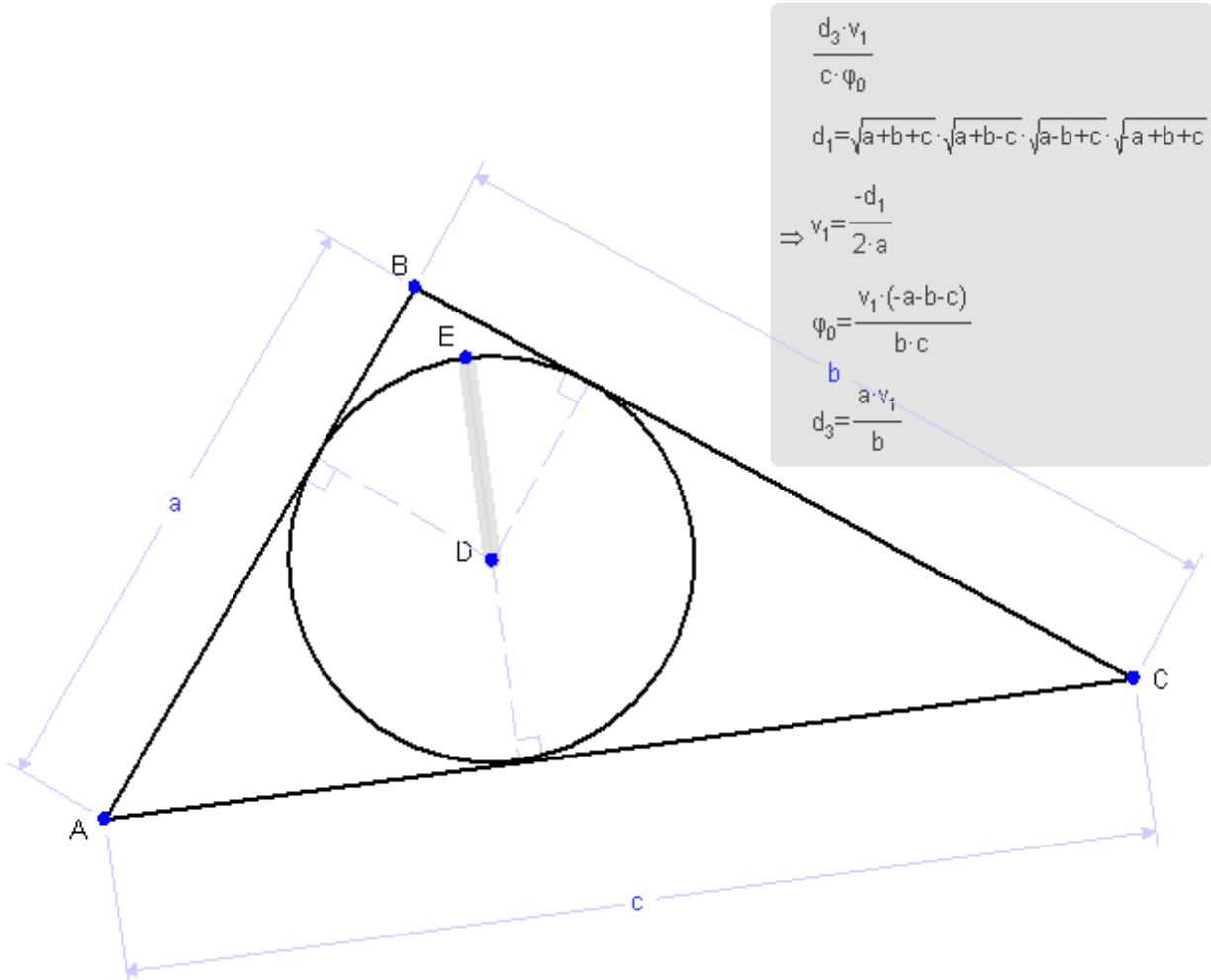
For example:



Becomes

$$[\sqrt{r-s}\,\sqrt{r+s}, 0 < r-s, 0 < 2\,r-2\,s]$$

- An expression with intermediate variables is converted into a list, with the main expression as the head of the list and the intermediate variable definitions following.



Becomes:

$$\left[\frac{v_1\, d_3}{\varphi_0\, c}, d_1 = \sqrt{-a+b+c}\,\sqrt{a-b+c}\,\sqrt{a+b-c}\,\sqrt{a+b+c}, v_1 = -\frac{d_1}{2\,a}, \varphi_0 = \frac{(-a-b-c)\,v_1}{c\,b}, d_3 = \frac{v_1\,a}{b}\right]$$

Working with Maple

Geometry Expressions gives the option of exporting expressions in Maple Input format: use

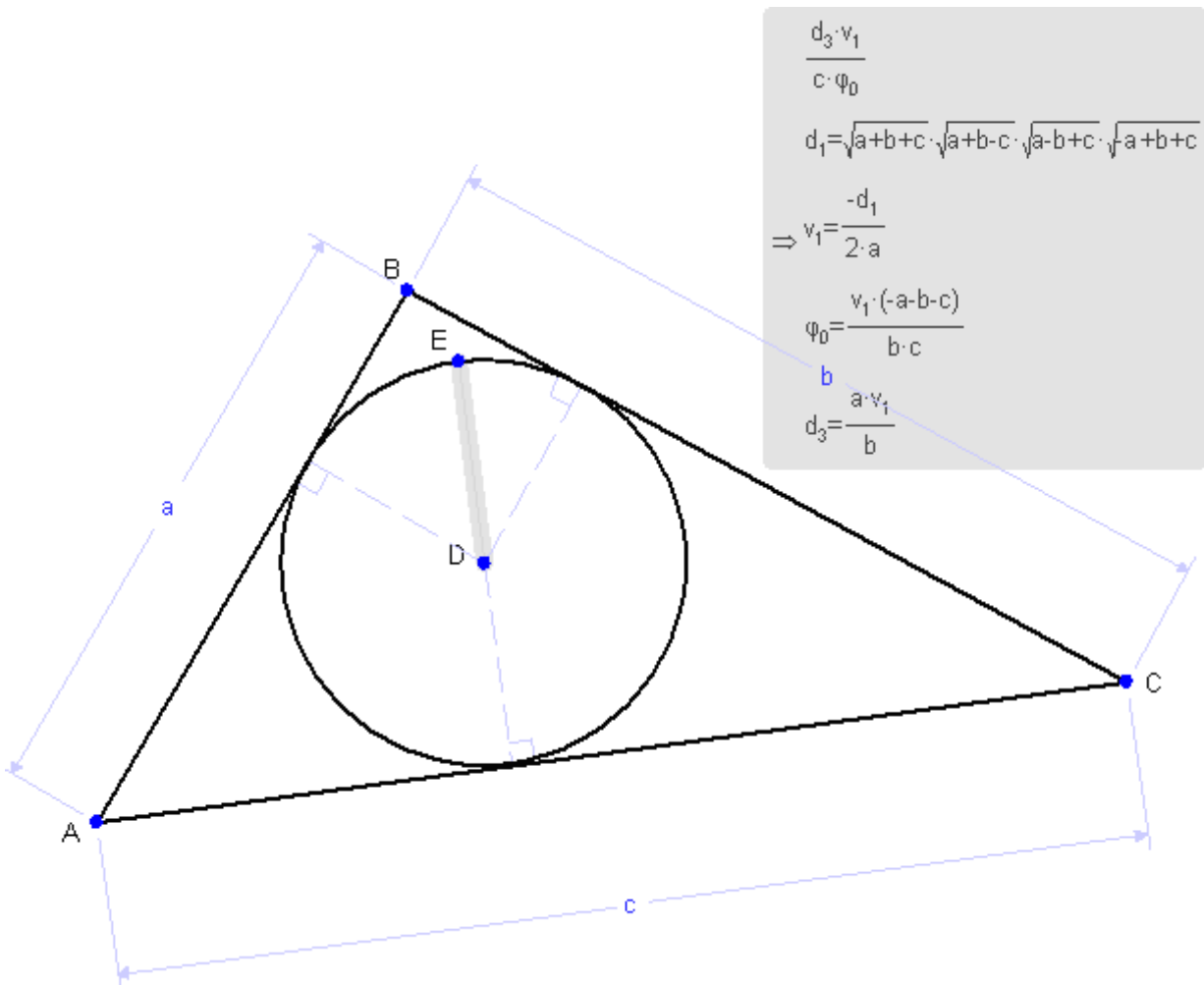
Copy As > Maple Input

This is the most reliable way of transferring expressions directly into Maple.

When you are using this option, expressions copy in quite a different form when intermediate variables are present and when they are not present. When intermediate

variables are present the expression copies as a Maple function, where intermediate variables are locals to the function. When intermediate variables are not present, a single Maple expression is created:

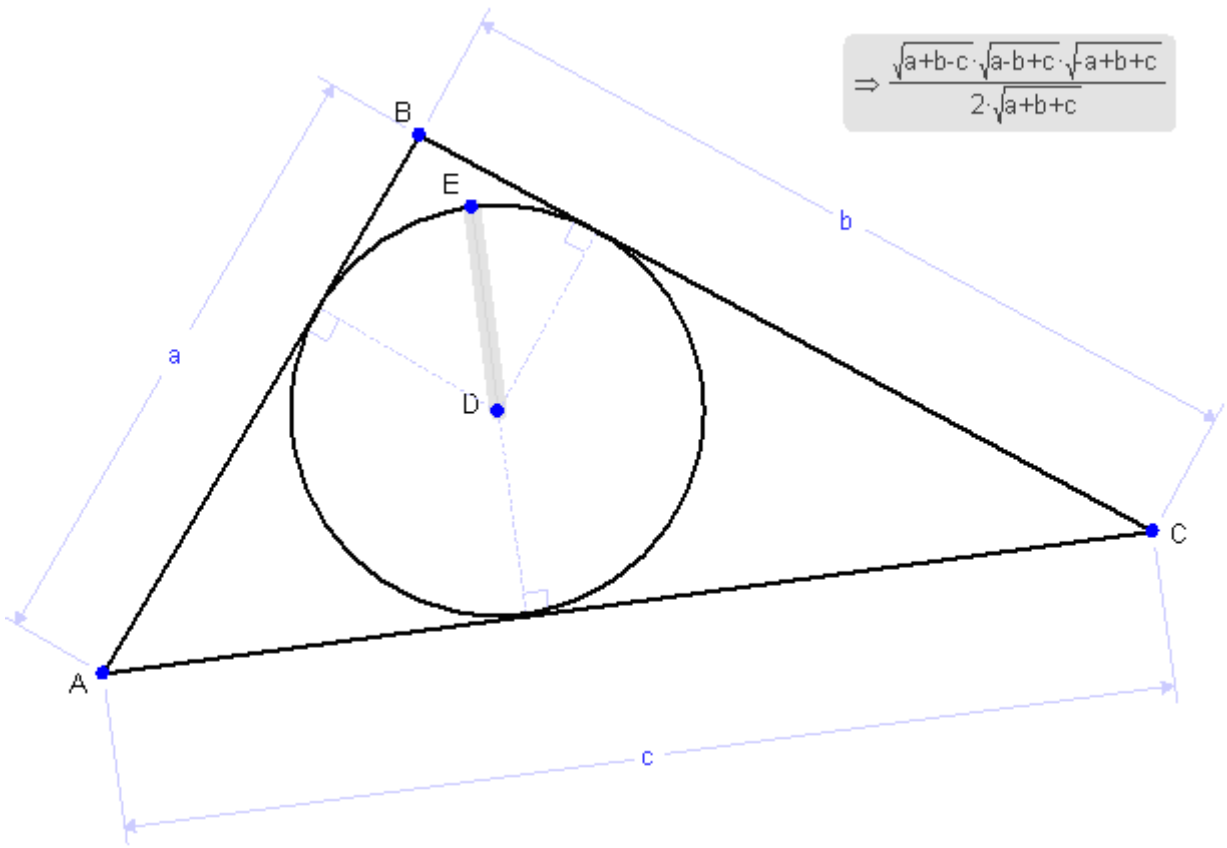
For example:



Becomes:

```
proc(a, b, c)
local d_1, v_1, phi_0, d_3;
    d_1 := (-a + b + c)^(1/2)*(a - b + c)^(1/2)*(a + b - c)^(1/2)*(a + b + c)^(1/2);
    v_1 := - (d_1)/(2*a);
    phi_0 := ((-a - b - c)*v_1)/(c*b);
    d_3 := (v_1*a)/(b);
    (v_1*d_3)/(phi_0*c);
end proc;
```

Whereas



Becomes

$$\frac{\sqrt{-a+b+c}\,\sqrt{a-b+c}\,\sqrt{a+b-c}}{2\,\sqrt{a+b+c}}$$

Working with Mathematica

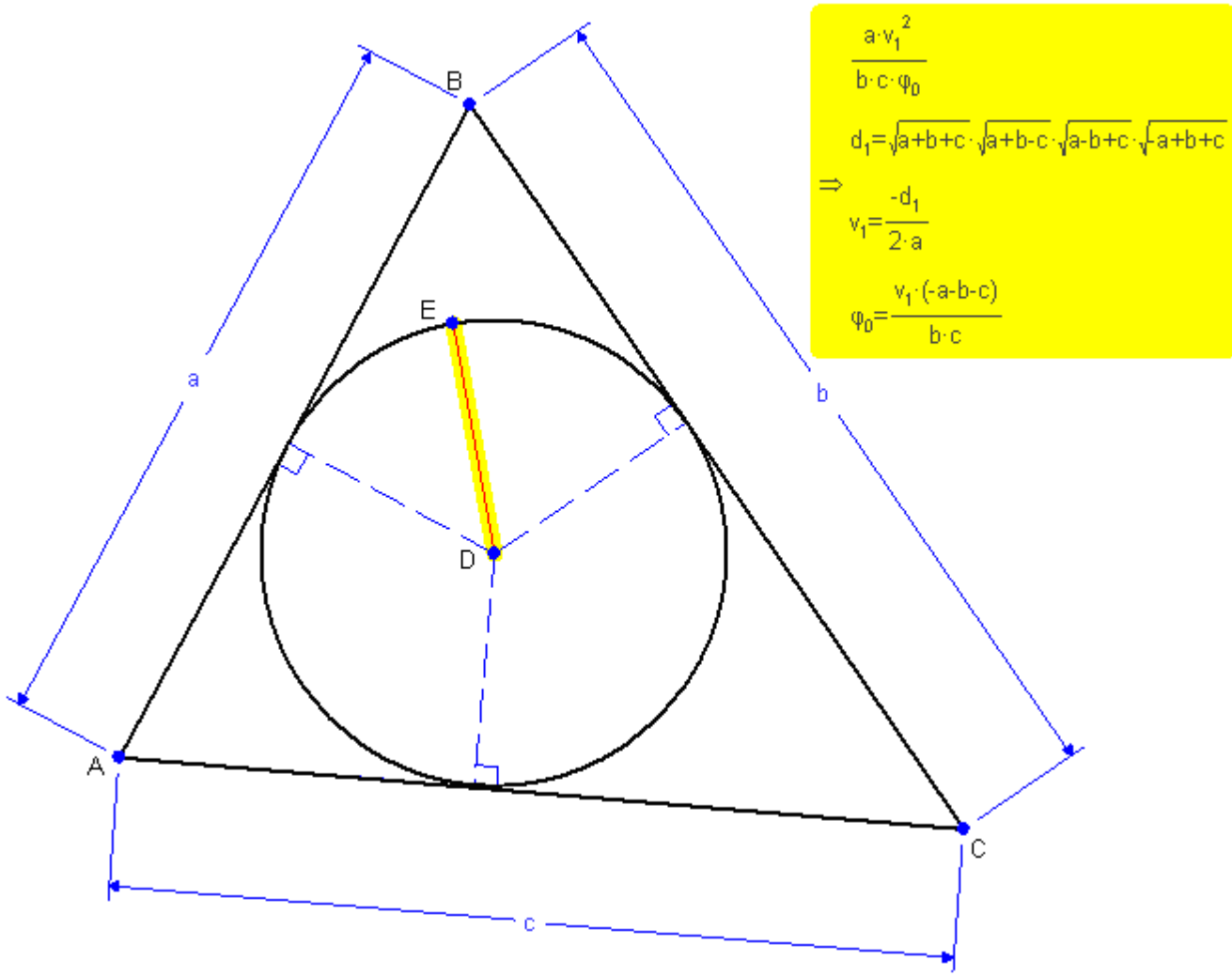
Geometry Expressions gives the option of exporting expressions in Mathematica Input format: use

Copy As > Mathematica Input

This is the most reliable way of transferring expressions directly into Mathematica.

When you are using this option, expressions copy in quite a different form when intermediate variables are present and when they are not present. When intermediate variables are present the expression copies as a Mathematica Module, where intermediate variables are locals to the module. When intermediate variables are not present, a single Mathematica expression is created:

For example,

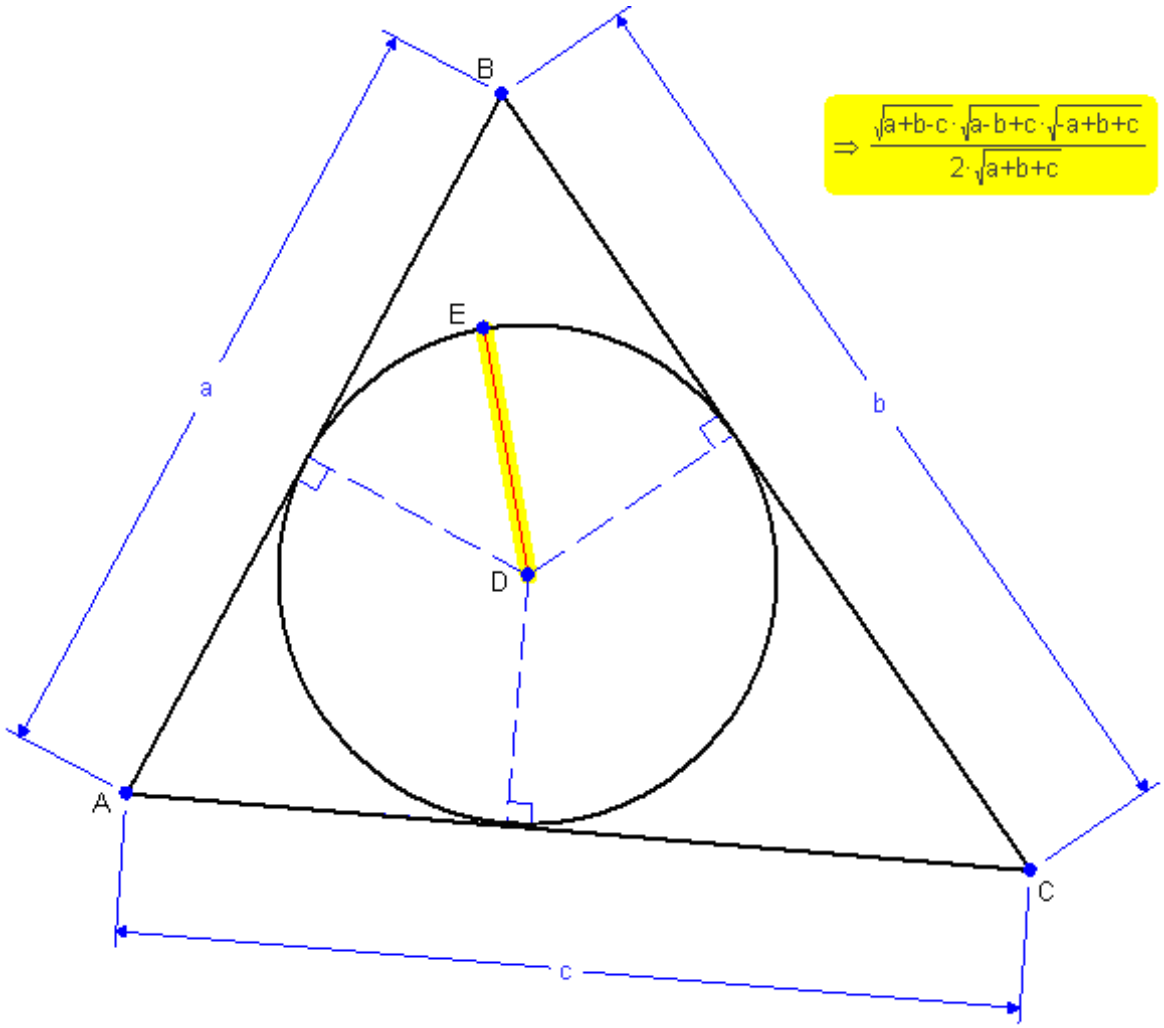


Copies as:

```
Module[{d$$1,v$$1,φ$$0},d$$1=((((a*(-1))+b+c))^(1/2)*((a+(b*(-
1))+c))^(1/2)*((a+b+(c*(-
1))))^(1/2)*((a+b+c))^(1/2));v$$1=(d$$1*(a)^((-1))*(-
1/2));φ$$0=(((a*(-1))+(b*(-1))+(c*(-1)))*v$$1*(c)^((-1))*(b)^((-
1)));((φ$$0)^((-1))*(v$$1)^(2)*(c)^((-1))*(b)^((-1))*a)]
```

$$-\frac{\sqrt{a+b-c}\,\sqrt{a-b+c}\,\sqrt{-a+b+c}\,\sqrt{a+b+c}}{2\,(-a-b-c)}$$

While:



Becomes:

```
((((a*(-1))+b+c))^(1/2)*((a+(b*(-1))+c))^(1/2)*((a+b+(c*(-
1))))^(1/2)*((a+b+c))^((-1/2))*1/2)
```

$$\frac{\sqrt{a+b-c}\,\sqrt{a-b+c}\,\sqrt{-a+b+c}}{2\sqrt{a+b+c}}$$

When you copy an expression as a module, you can work with the module numerically in Mathematica (by compiling it) without ever having to do a complete symbolic evaluation. This can be a useful feature when dealing with complicated expressions.

Copying from Mathematica into Geometry Expressions

In order to get Mathematica to create Content MathML, in a form suitable for Geometry Expressions, you should execute the following commands:

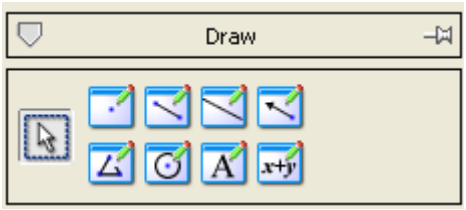
```
SetOptions[XML`MathML`ExpressionToMathML, "NamespacePrefixes" -> {},
"Formats" -> {"ContentMathML"}]

SetOptions[XML`MathML`BoxesToMathML, "NamespacePrefixes" -> {},
"Formats" -> {"ContentMathML"}]

SetOptions[Export, ConversionOptions -> {"NamespacePrefixes" -> {},
"Formats" -> {"ContentMathML"}}]

SetOptions[ExportString, ConversionOptions -> {"NamespacePrefixes" -> {},
"Formats" -> {"ContentMathML"}}]
```

Drawing Tools



The **Draw** toolbox contains the drawing modes and the Selection Arrow. The drawing modes can also be invoked from the **Draw** menu. Once selected, a mode is active until you change to a different one. You can find the active mode by noting which button is pressed or looking at the mode area in the status bar.

Links to the drawing modes are detailed below:


Point


Line Segment


Infinite Line


Vector

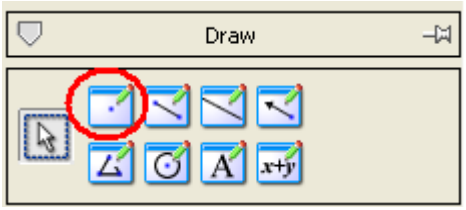

Polygon


Circle


Text

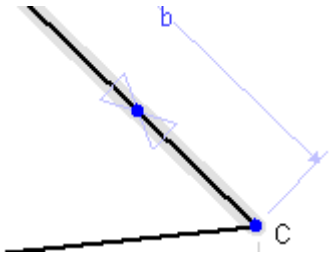

Expression

Add a Point

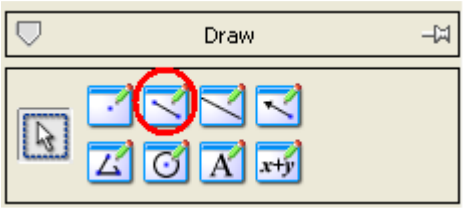


To insert a point in your drawing, follow these steps:

1. Click the **Point** icon  in the **Drawing** toolbox or select **Point** from the **Draw** menu.
2. Move the crosshairs  into position.
3. Click the mouse to place the point under the crosshair. When the crosshairs are positioned over some geometry an incidence symbol (bowtie) is displayed around the point and the geometry is highlighted. A click of the mouse will create the point incident to the geometry.



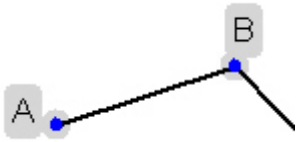
The Line Segment Tool



To add line segments to your geometry follow these steps:

1. Click on the **Line Segment** icon  in the **Draw** toolbox or select **Line Segment** from the **Draw** menu.
2. Position the cursor in the drawing window.
3. Click the mouse to place each endpoint.

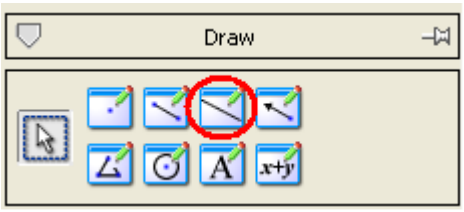
Each line segment is displayed with a letter label for each endpoint.



You can change the label from the Select mode  .

To abort a line segment in the middle of the drawing operation, hit the "esc" key.

Drawing Lines

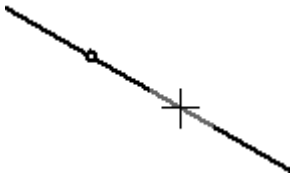


Lines are similar to line segments except they have infinite length.

1. Click the **Infinite Line** icon  in the **Draw** toolbox or select **Infinite Line** from

the **Draw** menu.

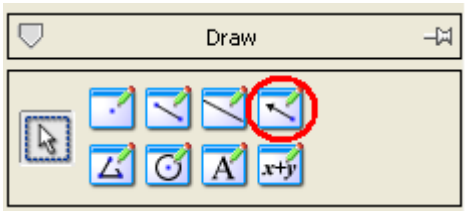
2. Position the line cursor in the drawing window.
3. Click the cursor to anchor the line at the cursor position. The anchor point will be displayed on the line.



4. Move the cursor in the drawing window to position the line and click the cursor when you get the line in the desired orientation.

Lines are infinite and do not have points associated with them unless you specifically place one on the line.

Drawing Vectors

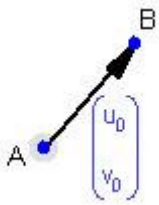


To add vectors to your geometry follow these steps:

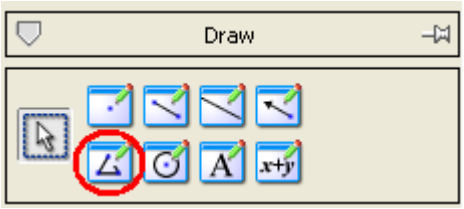
1. Click the **Vector** icon in the **Draw** toolbox or select **Vector** from the **Draw** menu.
2. Position the cursor in the drawing window.
3. Click the mouse to place each endpoint.

Each vector is displayed with a letter label for each endpoint.

Drawing vectors is similar to drawing line segments, but vectors are constrained with coefficients of the form:



The Polygon Tool

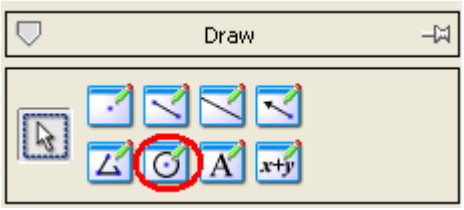


You can quickly create a multi-sided figure with these steps:

1. Click the **Polygon** icon in the **Draw** toolbox or select **Polygon** from the **Draw** menu.
2. Position the cursor in the drawing window.
3. Move the cursor and click once to place each vertex.

- As you create the sides of the polygon, each vertex is automatically assigned a letter name. You can change the label in Select mode.
- When you create the last side of the polygon by clicking on the first vertex, the polygon will be filled.
- To change the appearance of the polygon, select it and click the **Geometry** tab in the **Edit** / **Settings** menu.

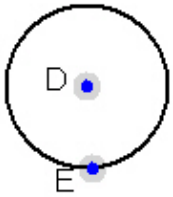
Drawing Circles



To add a circle to your diagram, follow these steps:

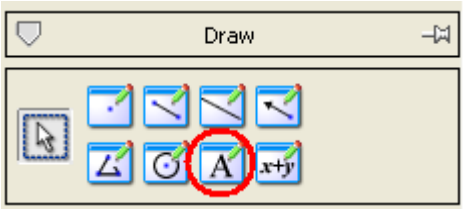
1. Click the **Circle** icon in the **Draw** toolbox or select **Circle** from the **Draw** menu.
2. Move the cursor in the drawing window to the position of the center of the circle and click once.
3. Move the cursor to draw the circle in the desired size and click again.

Notice the circle is displayed with 2 points, the center and a point on the perimeter.



You can adjust the circle in Select mode.

Adding Text to the Drawing

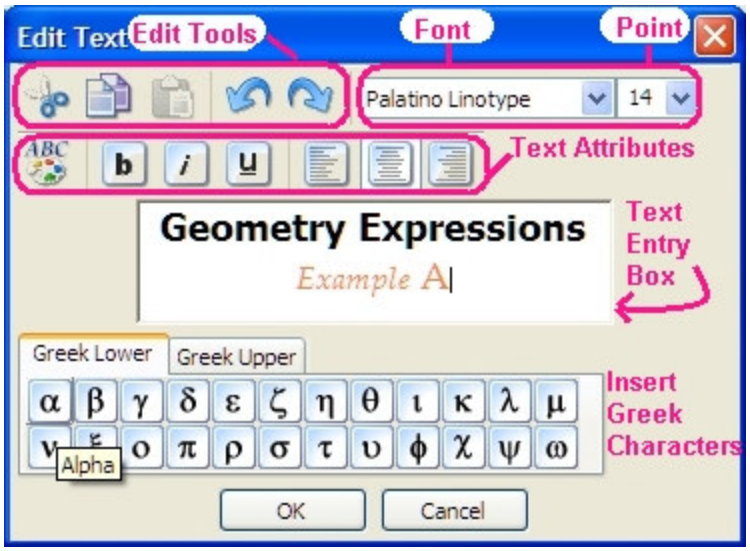


To add titles or other annotation to the drawing follow these steps:

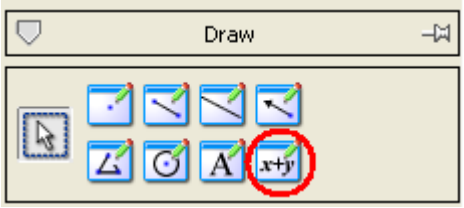
1. From the **Draw** toolbox click the **Text** icon or select **Text** from the **Draw** menu.
2. Position the text cursor at the upper left corner where you would like your window of text located.
3. Click and drag to form your text box.
4. Enter and format your text in the **Edit Text** dialog.

Inserting and Editing Text

In the **Edit Text** dialog you can enter and format the text that will be displayed in your defined text window.



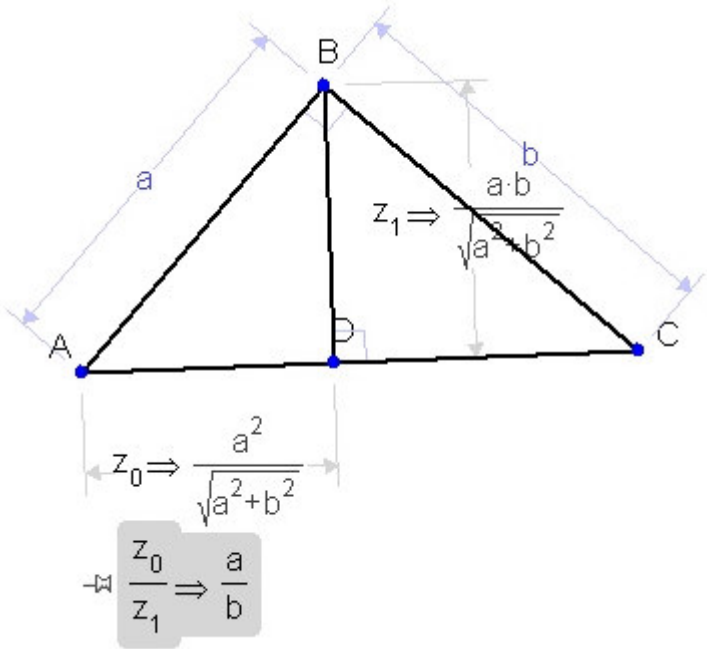
Adding Expressions to the Drawing



You can type an algebraic expression in the drawing window and Geometry Expressions will solve it with whatever information it has available. Here are the steps:

1. From the **Draw** toolbox click the **Expression** icon or select **Expression** from the **Draw** menu.
2. Move the expression cursor to the position where you want it to appear in the drawing window and click to display the data entry box.
3. Enter the expression using numbers, variables, and output expression names. Use the Symbols toolbox to help you enter mathematics.

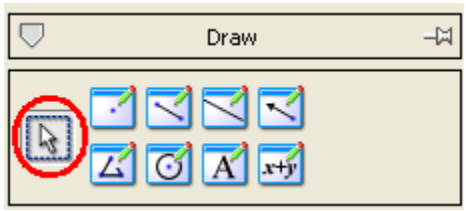
Adding an expression to the diagram can help you work out relationships in the geometry. Here is an example making use of the **Expression** tool to see the relationship between similar triangles.



Sides a and b are perpendicular, and AC is perpendicular to BD. We obtained the Output of lengths AD and BD using the Show Name button.

The expression to solve is the relationship of the two lengths: z_0 / z_1. The system immediately tells us => a/b.

The Selection Arrow

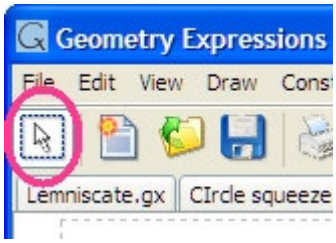


When you are finished with the drawing functions, click the selection arrow  to enable other functions or adjust your drawing.

You must first select elements of the drawing in order to enter constraints and constructions and to output calculations.

Many of the tools require you to select multiple objects.

The **Selection** arrow is also available form the icon bar at the top of the screen.



Selecting Multiple Objects

Many of the **Constrain**, **Construct**, and **Calculate** tools require that you select more than one object.

To select more than one object:

- Click the selection arrow on the tool bar.
- Hold down the CTRL or SHIFT key as you click the objects you want to select.

If you click the wrong object while holding down the CTRL key, click it again and it will become unselected.

If all of the objects you wish to select fall within a rectangular region, you can use the selection box:

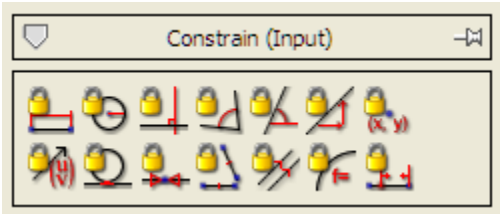
- Click on the selection arrow on the tool bar or the **Drawing** toolbox.
- Draw a rectangle around all of the objects by clicking and dragging.

Adjusting the Drawing

Click the Select arrow , either from the **Drawing** toolbox or the icon bar, to move, rotate or delete selected object(s) in the drawing.

You can also change a constraint value or label by double clicking it, retyping the value or variable and then pressing the enter key.

Drawing Constraints and How to Use Them



After sketching the geometry of a problem, constrain it with measurements, coordinates and implicit equations in real or symbolic terms. The drawing responds automatically to the assigned input constraints. Geometry Expressions will automatically add any constraints you leave out.

Initially, all the constraints in the toolbox are inactive. You must first select the parts for your drawing that will be constrained. Constraint choices are listed below along with the drawing elements that must be preselected. Be careful when selecting geometry objects, if extra things are selected that are not related to the constraint (like other constraints) the constraints will remain inactive. This can happen by mistake, especially when using the selection box tool.

	Constraint	**Preselected Objects**
	Distance / Length	Two of any combination of points, lines, line segments, vectors, or polygon sides.
	Radius	A circle
	Perpendicular	Two of any - lines, segments, vectors, or polygon sides.
	Angle	Two of any - lines, segments, vectors, or polygon sides.
	Direction	A line, segment, vector, or polygon side.
	Slope	A line, segment, vector, or polygon side.
	Coordinate	A point
	Coefficients	A vector
	Tangent	- A circle or locus and a line, segment, vector, or polygon side. - Or 2 circles
	Incident	A point and a line, segment, vector, polygon side, circle or locus.
	Congruent	Two or three of any line segment, vector, or polygon side.
	Parallel	Two or three of any line segment, vector, or polygon side.

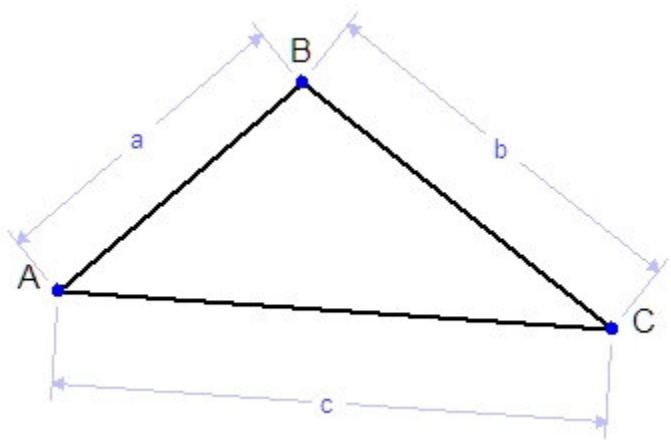
	Implicit Equation	A circle, line, segment, vector, or polygon side.
	Point Proportional Along Curve	A point and a line, segment, vector, polygon side, or locus

Occasionally you may try to add too many constraints to the geometry, causing a conflict. The system will help you correct this problem in the Resolve Constraint Conflict dialog.

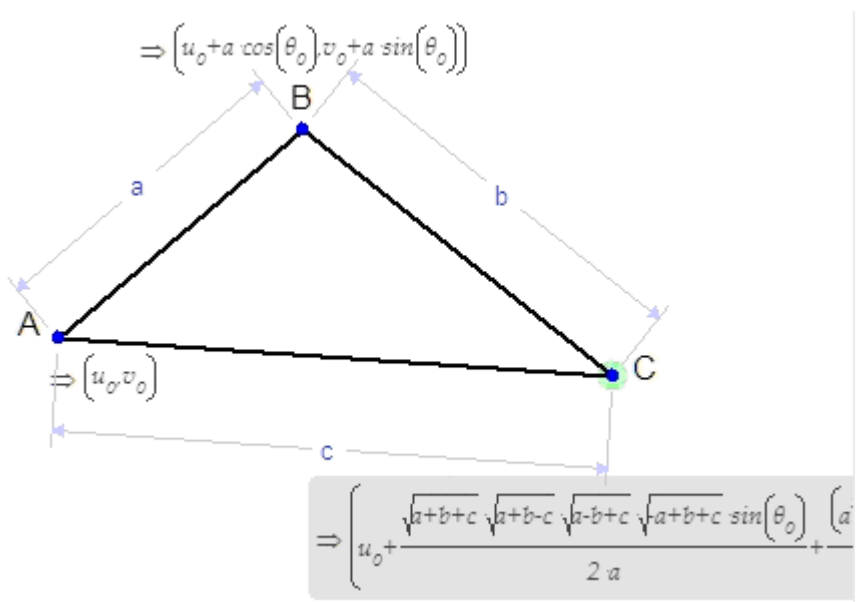
System Added Constraints

In Geometry Expressions, you do not need to fully constrain your model. Any parameters you have left unconstrained are filled in by the geometry engine.

For example, the following drawing is constrained only modulo a rigid transformation (typical for many geometry problems).



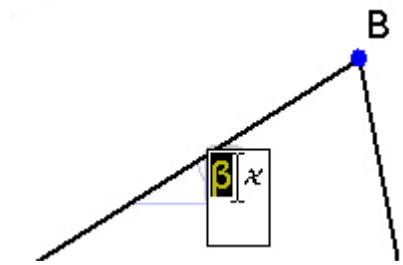
So if you ask for the coordinates of any of the points, they will be given in terms of system-added variables:



Clearly the system has added variables for the location of point A, and for the direction of line AB.

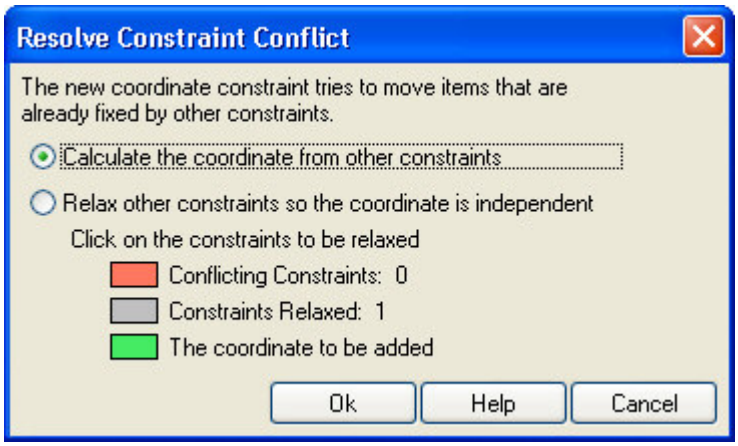
Changing a Constraint

To change a constraint, double click it with the selection arrow, retype the value or variable and press the enter key.



Constraint Conflicts

If you enter a constraint for some geometry which is already constrained by another constrained object you will see a message like the one below.



In this case, the **Coordinate** constraint was already determined by the other two sides and an angle constraint on the triangle. There are three ways of dealing with this problem:

Hit the **Cancel** button to leave the drawing as it was without the new constraint.

Click the first button, "Calculate the coordinates from other constraints" (the default choice). This is similar to the first choice, it eliminates the new constraint, but also calculates the selected geometry's value. In this case, the coordinates of point C would be calculated and displayed.

If you choose "Relax other constraints so the coordinate is independent", this will keep the new constraint you just entered and allow you to eliminate one of the red highlighted constraints (*figure 1*). When you select one of these constraints (in *figure 2* below we

clicked on θ), the highlight changes to gray. After you click **Ok**, the selected constraint, the angle θ in this case, is calculated and displayed (*figure 3*).

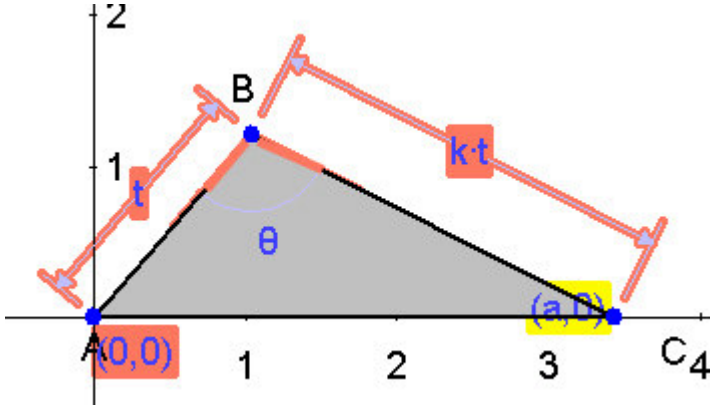


Figure 1

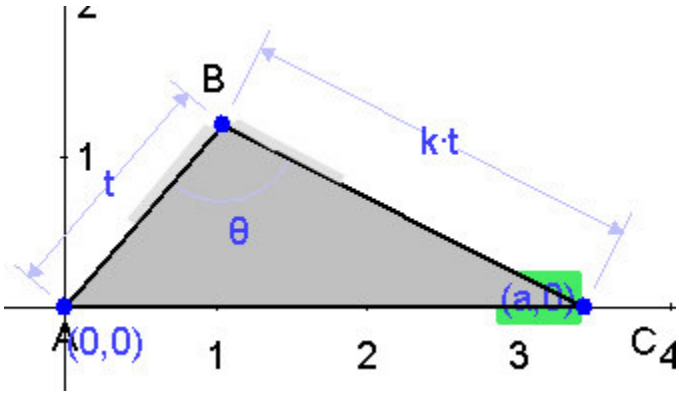


Figure 2

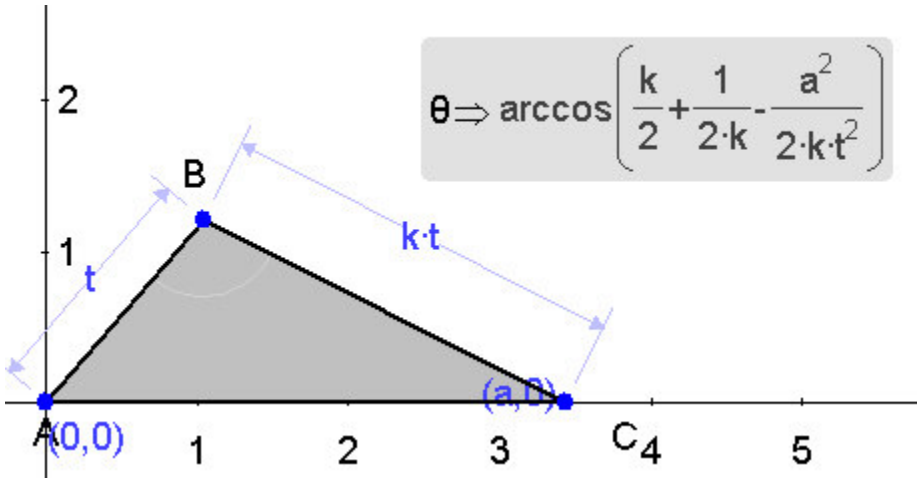
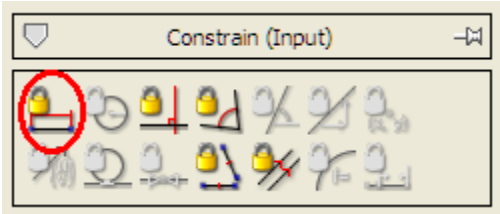


Figure 3

Distance / Length Constraint



The **Distance** / **Length** constraint lets you specify the following dimensions:

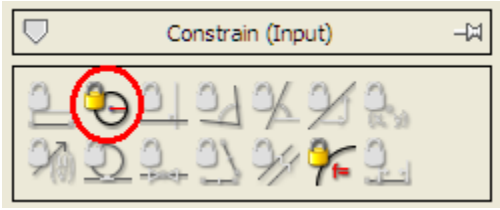
- Length of a line segment, vector, or polygon side
- Distance between two points or a point and any one of the line types listed above.

To enter a constraint:

- Select the appropriate drawing object(s). When you make your selection, the **Distance** / **Length** icon will light up .
- Click the icon, enter the constraint value, either real or symbolic, and press enter. You can press enter without typing a value to accept the system's default value.

You can click the constraint and drag it to adjust its placement on the drawing.

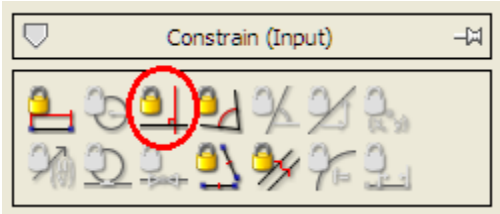
Radius Constraint



To specify the radius of a circle:

1. Make sure you're in select mode and click the circle. The circle will be highlighted as well as the icon .
2. Click the **Radius** icon, enter the constraint value, either real or symbolic, and press enter. You can press enter without typing a value to let the system insert a variable name.
3. You can click the constraint and drag it to adjust its placement on the drawing.

Perpendicular Constraint

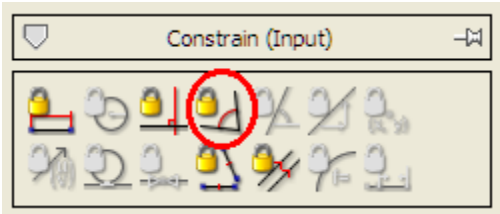


Any two of lines, segments, vectors or polygon sides can be constrained to be perpendicular with these steps:

1. Select two from the line types listed above.
2. Click the **Perpendicular** icon .

The lines are redrawn and the perpendicular constraint is attached.

Angle Constraint

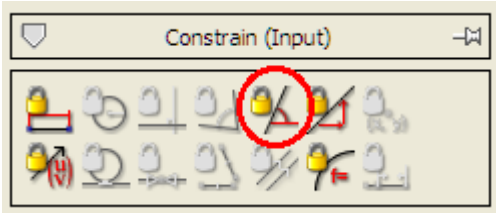


Any two of lines, segments, vectors or polygon sides can be constrained with an angle value or variable name with these steps:

1. Select two from the line types listed above.
2. Click the **Angle** icon .
3. Enter the constraint, real or symbolic. If you enter a real value, the lines will be adjusted to reflect the constraint.

Note: The angular units are displayed in the lower right of the screen. Change the default (Degrees or Radians) in the Edit / Settings menu.

Direction Constraint

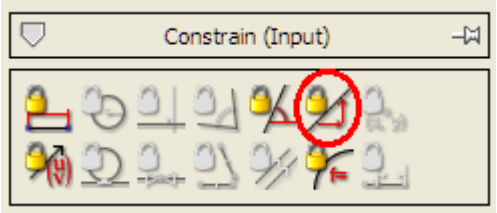


Constrain any of the line types; line, line segment, vector, or polygon side, to a direction measured from the horizontal.

1. Select one of the line types listed above.
2. Click the **Direction** icon .
3. Enter the constraint, real or symbolic. If you enter a real value, the line will be adjusted to reflect the constraint.

Note: The angular units are displayed in the lower right of the screen. Change the default (Degrees or Radians) in the Edit / Settings menu.

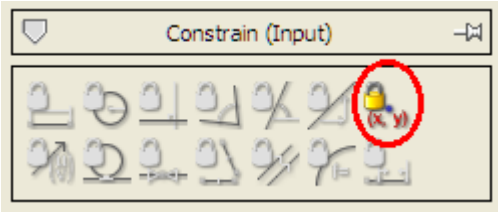
Slope Constraint



Specify a slope for any of the line types; line, line segment, vector, or polygon side.

1. Select one of the line types listed above.
2. Click the **Slope** icon .
3. Enter the constraint, real or symbolic. If you enter a real value, the line will be adjusted to reflect the constraint.

Coordinate Constraint

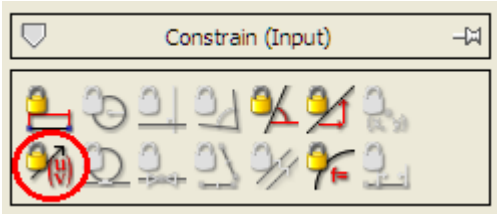


You can give coordinates to any point in your drawing:

1. Select a point.
2. Click the **Coordinate** icon .
3. Enter the constraint, real or symbolic. If you enter a real value, the line will be adjusted to reflect the constraint, even if the coordinate axes are not displayed.

To change the coordinates shown, double click and type over the highlighted value in the data entry box.

Constraining Vector Coefficients

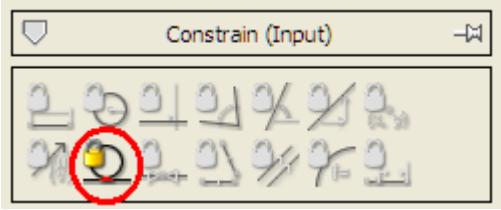


You can specify coefficients for a vector with the following steps:

1. Select a vector.
2. Click the coefficients icon .
3. Enter the coefficients separated by a comma.

Note: Don't forget the parentheses or an error message appears.

Tangent Constraint



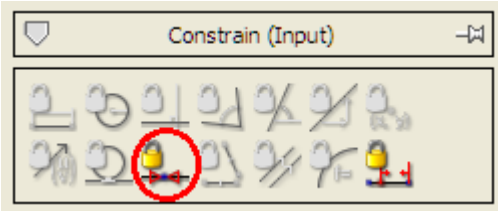
Any of the line types; line, line segment, vector, or polygon side can be made tangent to a circle or locus with these steps:

1. Select a line of the types listed above and the circle or locus.
2. Click the **Tangent** icon from the **Constrain** tool box or select **Tangent** from the **Constrain** menu.

The line and curve immediately become tangent.

You can select 2 circles and make them tangent.

Incident Constraint

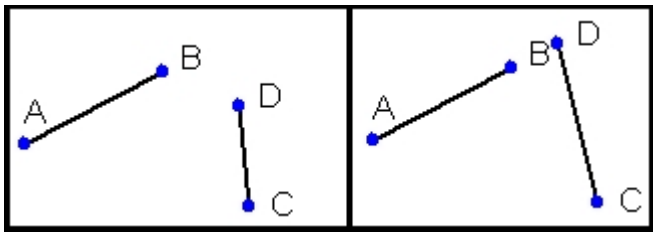


Constrain a point to be incident to any other geometry; line, segment, vector, polygon side, circle or locus with these steps:

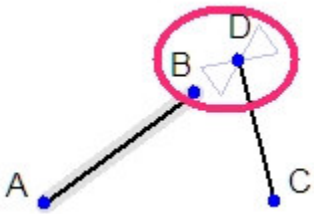
1. Select the point and the other geometry listed above.
2. Click the **Incident** icon from the **Constrain** toolbox, or select **Incident** from the **Constrain** menu.

The point is moved to meet the line or curve, or the extension of the line.

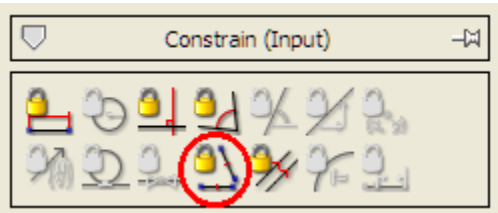
Below is an example of the latter, point D is moved to lie on the extension of line segment AB.



If you select the point or the line, incidence is indicated by a bowtie around the point:



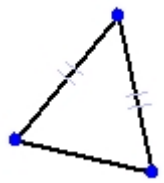
Congruent Constraint



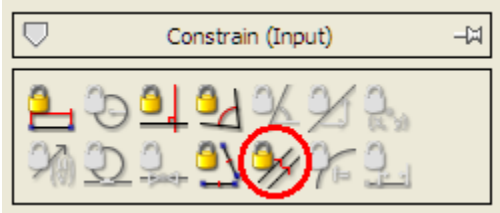
Constrain two or three of any of these geometry types: line segments, vectors, or polygon sides, to be congruent with these steps:

1. Select two line segments.
2. Click the **Congruent** icon from the **Constrain** toolbox, or select **Congruent** from the **Constrain** menu.

You will see matching congruency lines on the selected segments and a length will be adjusted.

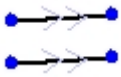


Parallel Constraint

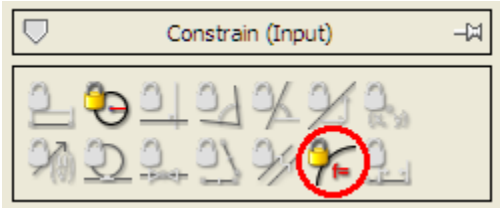


Any two or three of the linear geometry types can be made parallel - line, segment, vector, or polygon side.

1. Select two from the types listed above.
2. Click the **Parallel** icon from the **Constrain** toolbox, or select **Parallel** from the **Constrain** menu.

The geometry will be adjusted and matching symbols  appear on the selected lines.

Implicit Equation Constraint

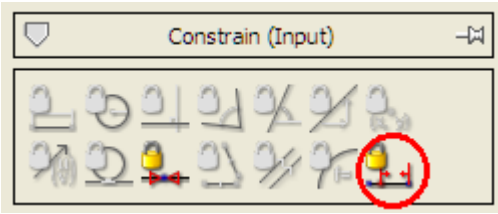


You can use symbolic variables to constrain geometry with an implicit equation. Lines, line segments, polygon sides, vectors and circles can all be constrained with implicit equations.

1. Select the geometry.
2. Click the **Implicit Equation** icon from the **Constrain** toolbox, or select **Implicit Equation** from the **Constrain** menu.

An input window will open next to the geometry you selected. Highlighted in the window is a generic equation for the selected object; for a line, an equation like – XA1+YB1+C1 = 0 might appear. You can edit the equation with different variable names or coefficients as you like. You will find these variables added to the variable list in the Variables toolbox.

Constraining a Point Proportionally Along a Curve

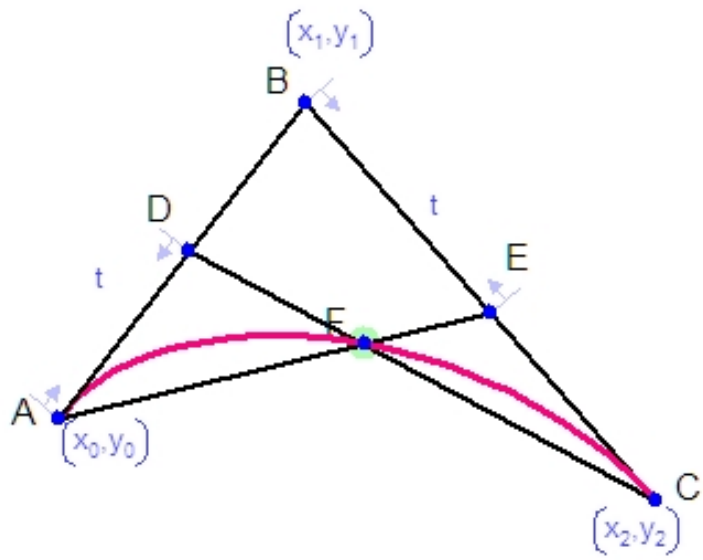


A point proportion t along a curve is defined variously for different types of curves as follows:

- For a line segment AB, it defines the point (1-t) A + tB
- For a circle it defines the point on the circle which subtends angle t at the center.
- For a locus or envelope, it defines the point at parameter value t.

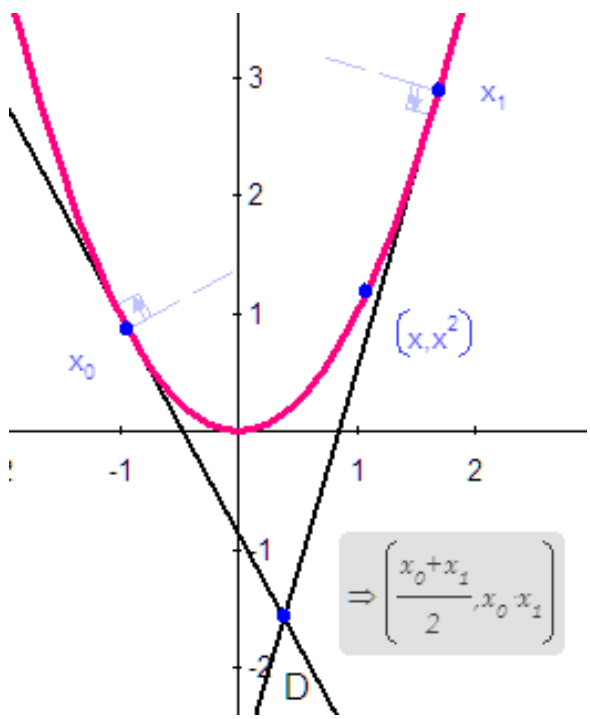
1. Select a point and one of the curves mentioned above.
2. Click the **Point Proportional…** icon from the **Constrain** toolbox, or select **Point Proportional…** from the **Constrain** menu.
3. Enter the parameter or quantity (symbolic or real) in the data entry box.

For example, in the following diagram, D is defined proportion t along AB, and E is defined proportion t along BC. The curve is the locus of F as t varies between 0 and 1.



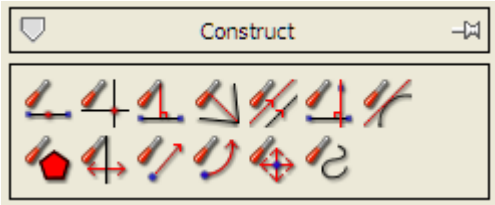
$$\Rightarrow \left(\frac{x_0-2\,t\,x_0+t^2\,x_0+t\,x_1-t^2\,x_1+t^2\,x_2}{1-t+t^2},\frac{y_0-2\,t\,y_0+t^2\,y_0+t\,y_1-t^2\,y_1+t^2\,y_2}{1-t+t^2}\right)$$

In the following example, the curve is the locus of the point (x, x^2). Tangents are created at points with parameter values x_0 and x_1 on this curve.



$$\Rightarrow \left(\frac{x_0+x_1}{2},x_0\,x_1\right)$$

Creating Constructions

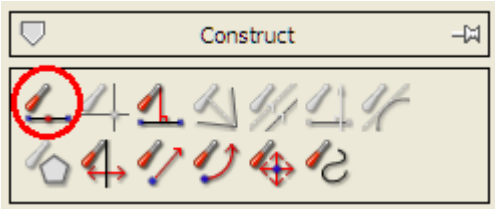


After sketching and constraining your drawing there are a whole set of constructions that can be applied to the geometry. First you must select the geometry elements which pertain to the construction. When you select the geometry the appropriate constructions will be highlighted.

The following table lists the **Constructions**, their icons, and which elements must be preselected to activate the constructions. Be careful when selecting geometry objects, if extra things are selected that are not related to the construction, the construction icons will remain inactive. This can happen by mistake, especially when using the selection box tool.

	Construction	**Preselected Objects**
	Midpoint	A line segment, vector, polygon side, or 2 points.
	Intersection	Two of: a line, segment, vector, or polygon side.
	Perpendicular Bisector	A line segment, vector, polygon side, or 2 points.
	Angle Bisector	Two of: a line, segment, vector, or polygon side.
	Parallel	A point and one of: a line, segment, vector, or polygon side.
	Perpendicular	A point and one of: a line, segment, vector, or polygon side.
	Tangent	A circle or constructed locus
	Polygon	- Three points or - Three or more line segments connected to form a polygon.
	Reflection	One or more objects
	Translation	One or more objects
	Rotation	One or more objects
	Dilation	One or more objects
	Locus	A point that will vary with a parameter

Constructing a Midpoint

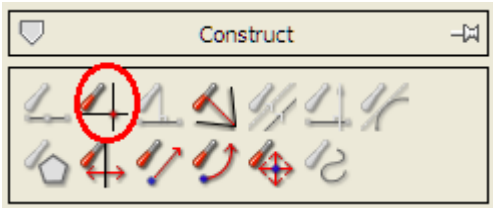


You can construct a midpoint on any line segment, vector, polygon side, or between two points by:

1. Select two points or one of the line types listed above.
2. Click the **Midpoint** tool, or select **Midpoint** from the **Construct** menu.

A point will appear in the middle of the selected line or 2 selected points.

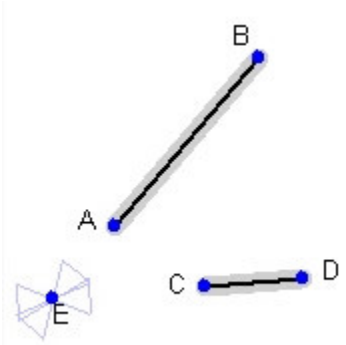
Intersections



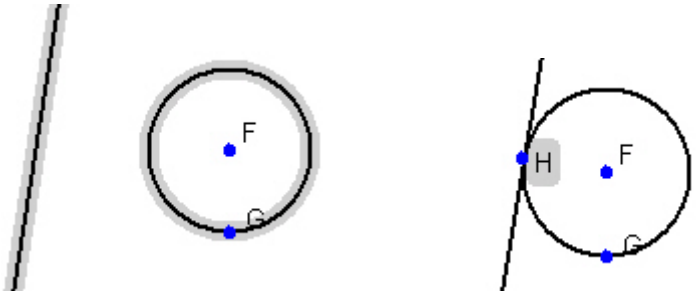
You can construct a point of intersection between any of the line types in your geometry; line, segment, vector, polygon side or circle.

1. Select two from the line types listed above.
2. Click the **Intersection** tool or select **Intersection** from the **Construct** menu.

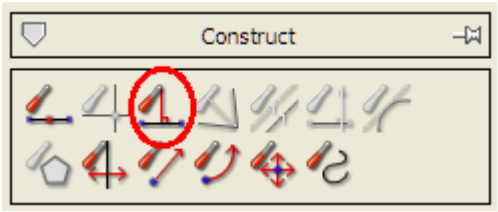
A new point and label will appear at the intersection. If the lines are segments that do not intersect, a point will be created at the extension of the lines as with line segments AB and CD below.



If the geometry will never intersect, the selected objects are moved to form the intersection. In the example below, the infinite line and circle become tangent at the newly created point, H.



Perpendicular Bisector



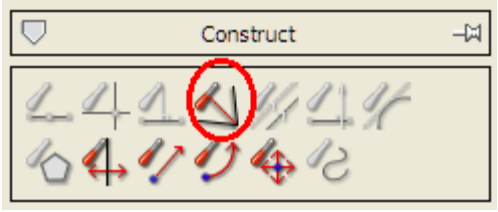
You can construct a perpendicular bisector on any line segment, vector, or polygon side with these steps:

1. Select any of the line types listed above.
2. Click the **Perpendicular Bisector** tool or select **Perpendicular Bisector** from the **Construct** menu.

An infinite line will appear at right angles to the selected line.

You can also construct a perpendicular bisector by selecting 2 points.

Angle Bisector

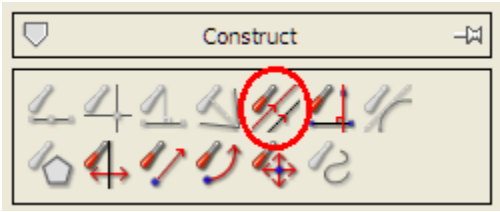


You can bisect the angle between any combination of line types; line, segment, vector, or polygon side with these steps:

1. Select two of any of the line types listed above.
2. Click the **Angle Bisector** tool or select **Angle Bisector** from the **Construct** menu.

An infinite line will appear between the two selected lines. You can use the Calculate / Angle tool to get the value of the bisected angle.

Parallel Constructions

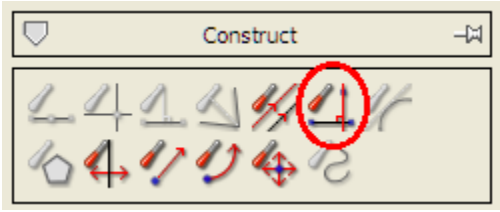


You can construct a line, through a point, and parallel to another line, segment, polygon side or vector with these steps:

1. Select a point and a line of one of the types listed above.
2. Click the **Parallel** tool or select **Parallel** from the **Construct** menu.

A line is constructed which is parallel to the selected line and passes through the selected point

Perpendicular Constructions

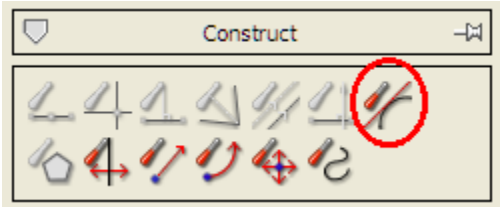


You can construct a line, through a point, which is perpendicular to another line, segment, polygon side or vector with these steps:

1. Select a point and a line of one of the types listed above.
2. Click the **Perpendicular** tool or select **Perpendicular** from the **Construct** menu.

A line is constructed which is perpendicular to the selected line and passes through the selected point

Tangents

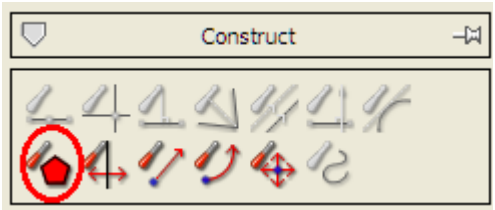


You can construct a line that is tangent to a circle or locus with these steps:

1. Select the circle or locus.
2. Click the **Tangent** tool or select **Tangent** from the **Construct** menu.

A line tangent to the selected curve will appear at the point where you selected the circle or curve.

Polygon Construction

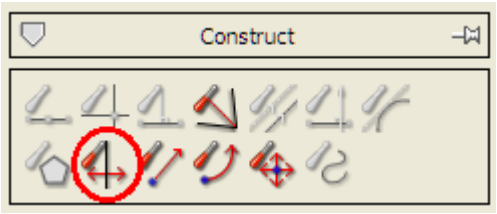


If you created a polygon with the line segment tool, or your polygon was not shaded for some reason, (*e.g.* the drawing of the sides was interrupted or out of order) you can make joined line segments into a polygon that can be selected with a single click using this construction.

1. Select the line segments that make up the polygon or three points.
2. Click the **Polygon** tool in the **Construct** toolbox, or select **Polygon** from the **Construct** menu.

The polygon will be filled and you can now select the entire polygon with a single click.

Reflection



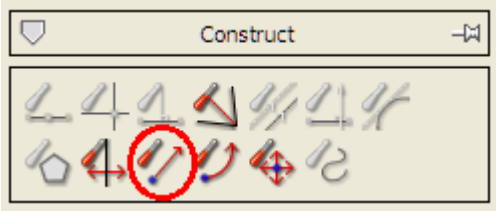
You can reflect any subset of your diagram about a line with these steps:

1. Select one or more geometry objects to reflect.
2. Click the **Reflection** tool in the **Construct** toolbox, or select **Reflection** from the **Construct** menu.
3. Either click the cursor to place the reflection line on the screen, adjust the angle and click again, or select an existing line as the reflection line.

A copy of your selected geometry will appear on the other side of the reflection line.

Notice all points on the reflected geometry are written as "prime", *i.e.* A becomes A′. If you reflect the geometry again, A′ becomes A".

Translation



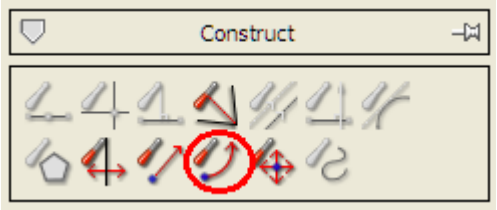
You can translate any subset of your diagram with a translation vector. Here are the steps:

1. Select the geometry to be translated.
2. Click the **Translation** tool in the **Construct** toolbox, or select **Translation** from the **Construct** menu.
3. Click the cursor to draw the end point of your translation vector and move the cursor to establish the length and angle of the translation. Click again to finish the vector.

The translated geometry appears. You can adjust the position of the translation by clicking and dragging the tip of the vector.

Notice all points on the translated geometry are written as "prime", *i.e.* A becomes A'. If you translate this geometry again, A' becomes A".

Rotation



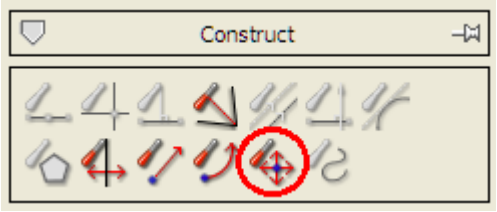
You can rotate any subset of your diagram about a point. Here are the steps:

1. Select the geometry to be rotated.
2. Click the **Rotation** tool in the **Construct** toolbox, or select **Rotation** from the **Construct** menu.
3. Click the screen to place your rotation point.
4. In the data entry box presented, enter the angle of rotation.

The rotation of the selected geometry appears.

Notice all points on the rotated geometry are written as "prime", *i.e.* A becomes A′. If you rotate this geometry again, A′ becomes A".

Dilation



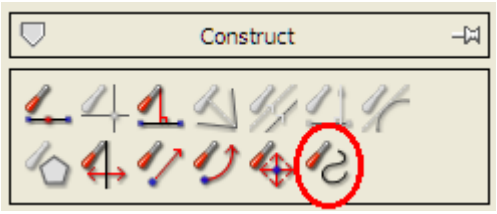
You can dilate any subset of your diagram from a point. Here are the steps:

1. Select the geometry to be dilated.
2. Click the **Dilation** tool in the **Construct** toolbox, or select **Dilation** from the **Construct** menu.
3. Click the screen to place your dilation point.
4. In the data entry box presented, enter the dilation factor.

The dilated geometry appears.

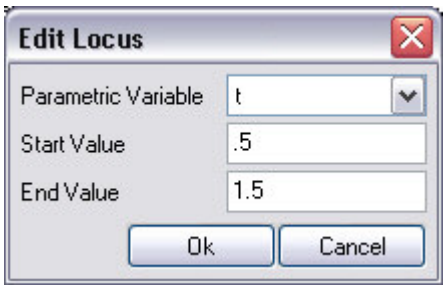
Notice all points on the dilated geometry are written as "prime", *i.e.* A becomes A′. If you dilate this geometry again, A′ becomes A".

Locus of Points / Envelope

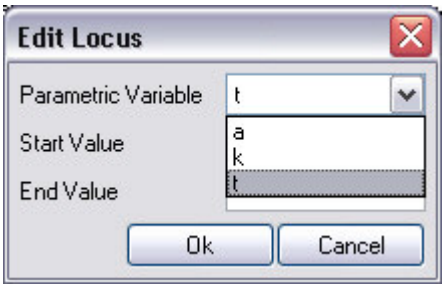


You can construct a locus of points or envelope from a selected point or line, by defining a range for some constraint in the drawing. Just follow these easy steps:

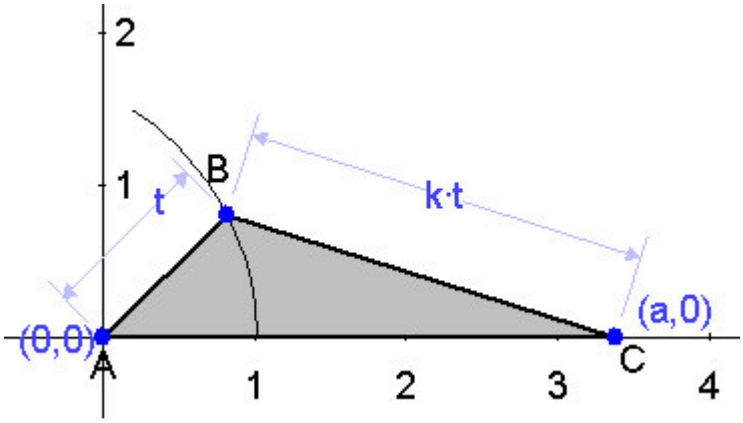
1. Select the point on the drawing that will form the locus (point B in the example below); select a line, line segment, or vector to form an envelope.
2. When you click the **Locus** icon , the **Edit Locus** dialog pops up.



3. Simply fill in the values for the appropriate variable. Click the arrow key to the right of the **Parametric Variable** window to select from a list of all variables in the drawing. (If you entered the constraint in real terms, change it to a variable by double-clicking the constraint.)

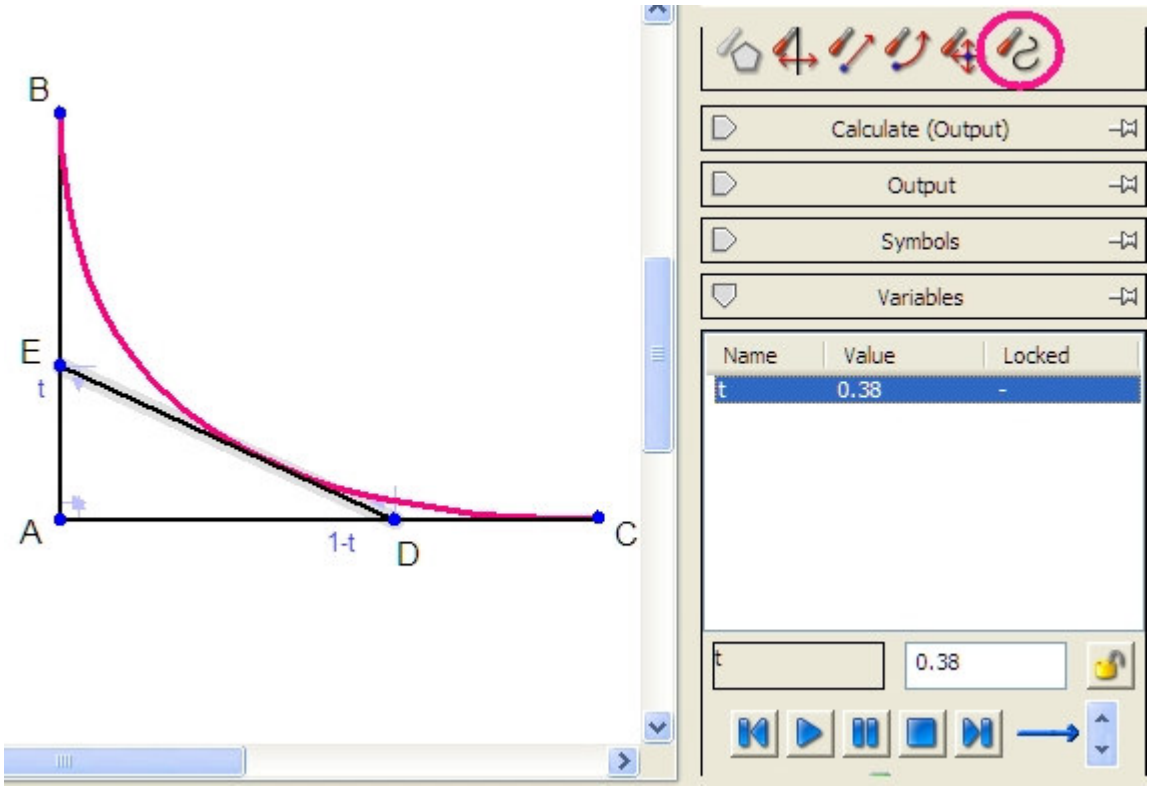


4. Click the **Ok** button and the locus will appear.



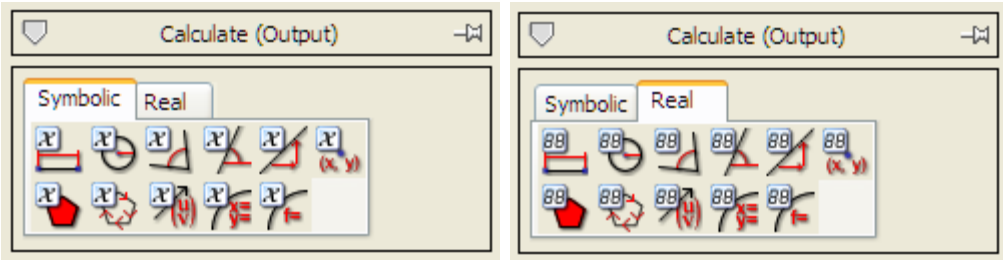
! Note: The locus only works if the figure's position is fixed - i.e. a point in a triangle will not work as a locus unless the locations of the other two points are fixed with coordinate constraints.

The following example shows an envelope of the line DE. We use the Point proportional along curve constraint and the parameter t to position the points D and E (D is (1-t) along line AC and E is t along line AB). In the **Edit Locus** dialog, we create the envelope from parameter t as it ranges from 0 to 1.



You can replay the creation of the envelope with the Animation tools. Select variable t and click the Play button.

Output Calculations



Geometry Expressions will make calculations in the geometry based on any constraints or constructions you have specified, or just from the sketch. Calculations can be output in **Real** or **Symbolic** terms by choosing the appropriate tab.

If you haven't supplied all of the necessary input constraints, the system inserts any missing variables automatically or, for real calculations, bases the value on the sketch.

Some calculations may take longer than you expected. If the calculation isn't immediate, you can stop it and restart it with these buttons on the tool bar.

 Stops calculations

 Restarts calculations.

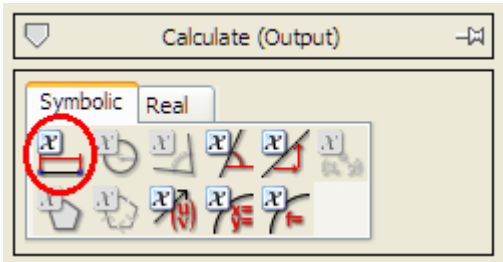
You may want to reconsider the problem, add constraints, or just let it continue with the calculation.

The table below lists all the available calculations and geometry elements which must be preselected. Be careful when selecting geometry objects, if extra things are selected that are not related to the calculation (like other calculations) the calculations will remain inactive. This can happen by mistake, especially when using the selection box tool.

	Calculation	**Preselected Object(s)**
	Distance / Length	A line segment, vector, or polygon side, or a point and one of these line types (perpendicular distance), or two points.
	Radius	Circle
	Angle	Two of any line, segment, vector, or polygon side.
	Direction	A line, segment, vector, or polygon side.
	Slope	A line, segment, vector, or polygon side.
	Coordinates	Point
	Area	Circle or polygon

	Perimeter	Circle or polygon
	Coefficients	Vector
	Parametric Equation	A line, segment, vector, polygon side, circle, or a constructed locus.
	Implicit Equation	A line, segment, vector, polygon side, circle, or a constructed locus.

Distance / Length Calculation

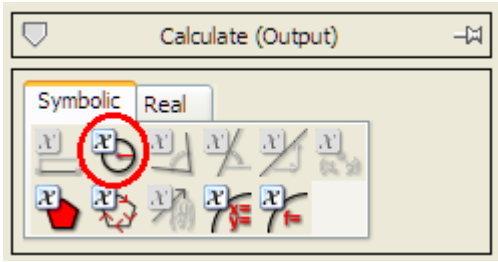


Length calculations may be obtained for any line segment, polygon side or vector. Distance calculations are available between two points, or the perpendicular distance between a point and a line, segment, polygon side, or vector. Use these steps:

1. Select the line segment, or point and line, or pair of points as described above.
2. Click the tab to switch from Real to Symbolic output or *vice versa.*
3. Click the **Distance** / **Length** tool in the **Calculate** toolbox or select **Distance** / **Length** from the **Calculate** menu.

Geometry Expressions displays the length, using any relevant parameters you may have specified.

Radius Calculation

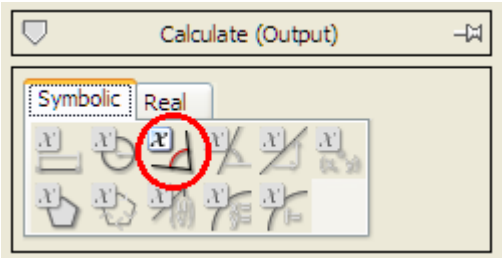


Geometry Expressions will calculate the radius of any circle. Use these steps to find the radius:

1. Select a circle.
2. Click the tab to switch from Real to Symbolic output or *vice versa.*
3. Click the **Radius** tool in the **Calculate** toolbox or select **Radius** from the **Calculate** menu.

The equation with real or symbolic terms appears in the diagram.

Angle Calculation



Geometry Expressions will calculate any angle between lines in the geometry. Use these steps to find the angle:

1. Select two line types.
2. Click the tab to switch from Real to Symbolic output or *vice versa.*
3. Click the **Angle** tool in the **Calculate** toolbox or select **Angle** from the **Calculate** menu.

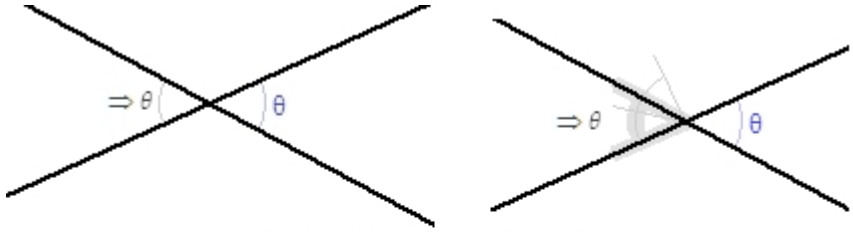
The equation with real or symbolic terms appears in the diagram.

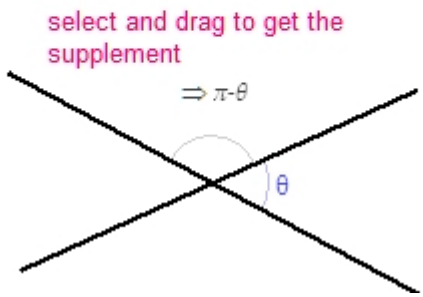
You can obtain the angle's supplement by dragging the angle symbol.

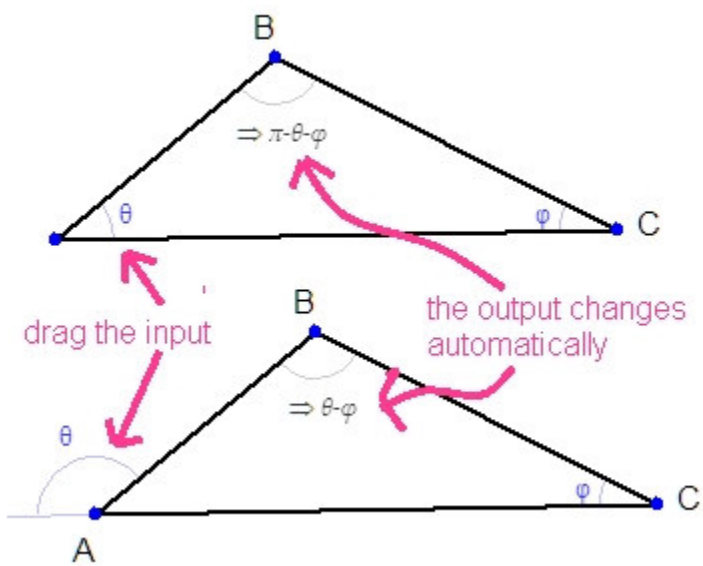
Supplementary Angles

If it's unclear whether a calculation is requested for the angle or its supplement, you can drag the angle symbol to the correct position.

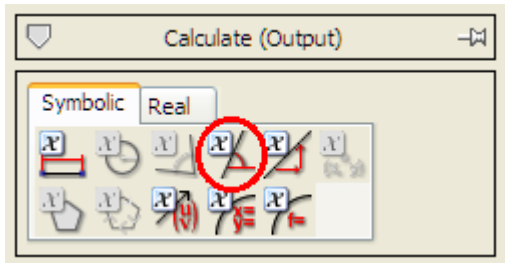
Here are some examples of playing around with supplementary angles, both inputs and outputs:







Direction Calculation

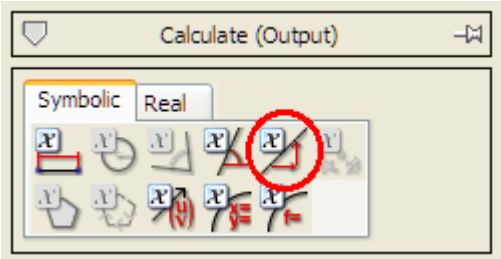


Geometry Expressions will calculate the direction of lines, segments, polygon sides, or vectors with these steps:

1. Select a line type.
2. Click the tab to switch from Real to Symbolic output or *vice versa.*
3. Click the **Direction** tool in the **Calculate** toolbox or select **Direction** from the **Calculate** menu.

The direction measurement appears in real or symbolic terms.

Slope Calculation

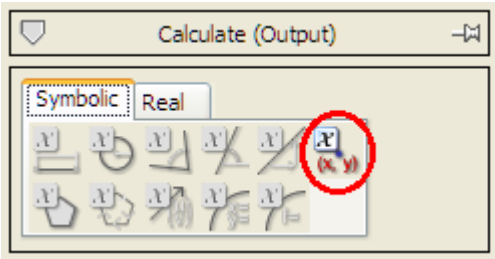


Geometry Expressions will calculate the slope of lines, segments, polygon sides, or vectors with these steps:

1. Select a line type.
2. Click the tab to switch from Real to Symbolic output or *vice versa.*
3. Click the **Slope** tool in the **Calculate** toolbox or select **Slope** from the **Calculate** menu.

The equation for the slope with real or symbolic terms appears in the diagram.

Calculate Coordinates



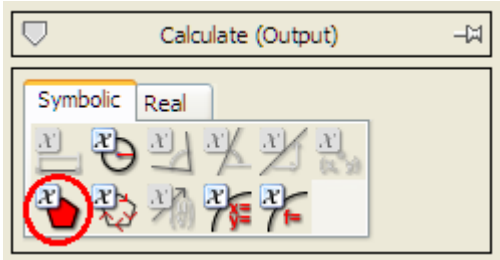
You can calculate the coordinates of any point in your diagram with these steps:

1. Select a point.
2. Click the tab to switch from Real to Symbolic output or *vice versa.*

3. Click the **Coordinates** tool in the **Calculate** toolbox or select **Coordinates** from the **Calculate** menu.

The real or symbolic coordinates appear by the point.

Area Calculation



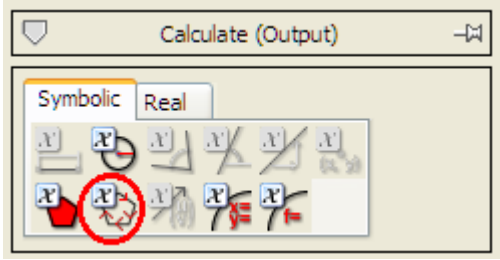
You can obtain the area of any polygon or circle in your diagram.

Note: If your polygon is not filled it is just a group of line segments. To convert them to a polygon, use the Polygon Construction tool, then proceed with these steps:

1. Select a circle or polygon.
2. Click the tab to switch from Real to Symbolic output or *vice versa.*
3. Click the **Area** tool in the **Calculate** toolbox or select **Area** from the **Calculate** menu.

The area is displayed in real or symbolic terms.

Perimeter Calculation



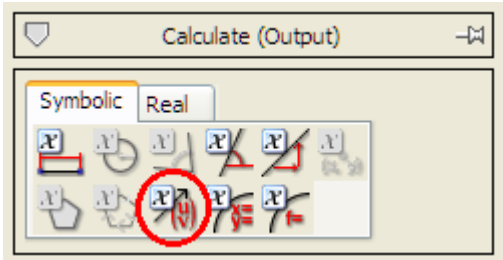
You can obtain the perimeter of any polygon or circle in your diagram.

Note: If your polygon is not filled it is just a group of line segments. To convert them to a polygon, use the Polygon Construction tool, then proceed with these steps:

1. Select a circle or polygon.
2. Click the tab to switch from Real to Symbolic output or *vice versa.*
3. Click the **Perimeter** tool in the **Calculate** toolbox or select **Perimeter** from the **Calculate** menu.

The perimeter is displayed in real or symbolic terms.

Calculate Coefficients

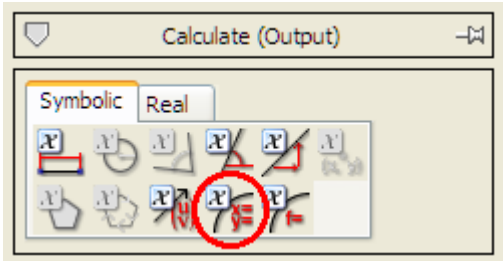


Use this tool to calculate the coefficients of a vector in the diagram with these steps:

1. Select a vector.
2. Click the tab to switch from Real to Symbolic output or *vice versa.*
3. Click the **Coefficients** tool in the **Calculate** toolbox or select **Coefficients** from the **Calculate** menu.

The real or symbolic coefficients appear by the point.

Calculating Parametric Equations



Computes parametric equations for a locus or envelope, based on the parameter defining the curve.

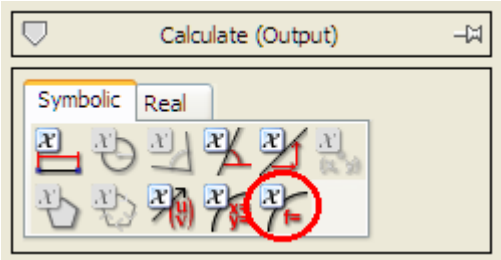
You can also calculate parametric equations for a circle or line.

Use these steps:

1. Select any geometry object described above.
2. Click the tab to switch from Real to Symbolic output or *vice versa.*
3. Click the **Parametric Equation** tool in the **Calculate** toolbox or select **Parametric Equation** from the **Calculate** menu.

The real or symbolic equations for x and y appear by the geometry.

Calculating Implicit Equations



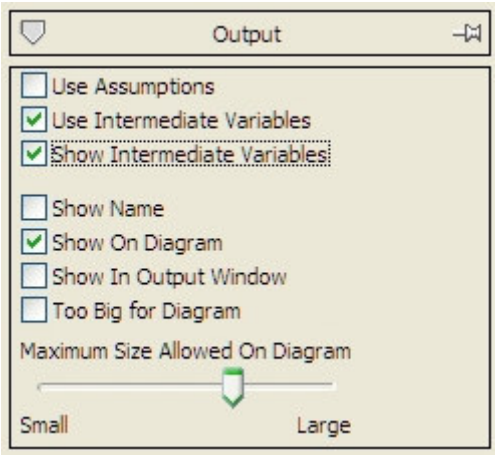
Calculates the implicit equation for the selected circle or a line.

Geometry Expressions will also attempt to calculate the equation of a locus or envelope curve.

1. Select any geometry object described above.
2. Click the tab to switch from Real to Symbolic output or *vice versa.*
3. Click the **Implicit Equation** tool in the **Calculate** toolbox or select **Implicit Equation** from the **Calculate** menu.

The real or symbolic equation appears by the geometry.

Viewing the Output



The **Output** tools apply to Symbolic Calculations.

After you have made a calculation, click the resulting output, either in the drawing window or the output window. This will highlight entries in the **Output** tool box to refine your calculation.

The **Output** check boxes specify how the calculations are made and how the output is displayed.

- Use Assumptions – applies to equations containing absolute values.
- Use Intermediate Variables – can sometimes simplify the output.

The "Show" check boxes control where the output is displayed.

Click a check box to show (when checked) or not show (when not checked).

Intermediate Variables

Substituting intermediate variables can sometimes simplify expressions calculated by the program. Try this option to see how it affects your output:

1. Select an output calculation.
2. Click the Use Intermediate Variables check box.
3. Click the box again to change back to the previous form.

If Use Intermediate Variables is not checked, the Show Intermediate Variables is inactive.

Note: Intermediate variables are not always used in calculations, in which case this box will have no effect.

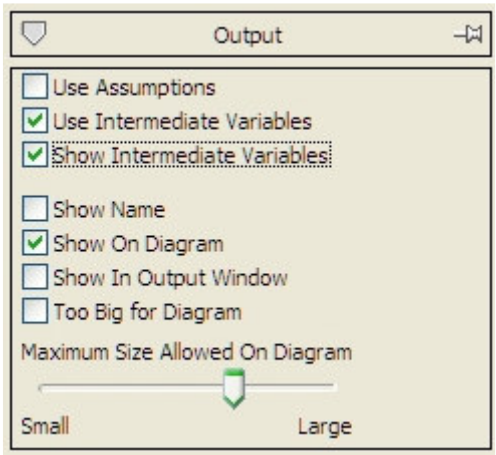
Assumptions

With the Use Assumptions check box, Geometry Expressions eliminates the absolute value from expressions and determines from the diagram whether the value is positive or negative. Try it with these steps:

1. Select an output calculation.
2. Click the Use Assumptions check box.
3. Click the box again to change back to the previous form.

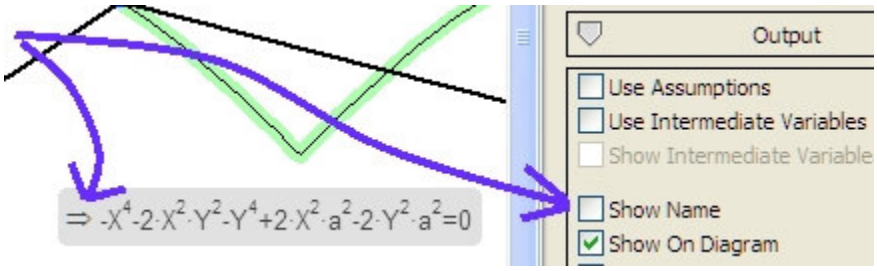
Obviously, this tool only has an effect when the expression selected contains an absolute value.

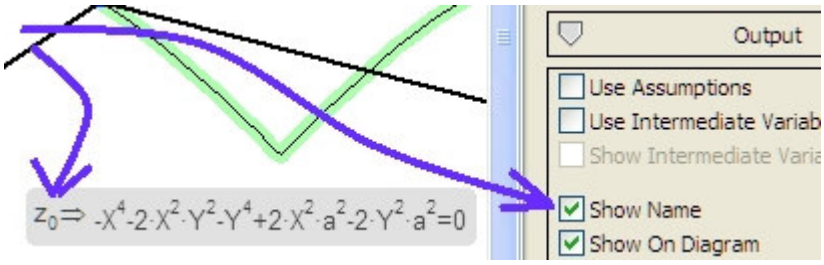
Show Output Check Boxes



After you generate an output expression:

1. click the output
2. click one or more of the **Output** Show. . . check boxes to apply them to the selected expression
   - Show Intermediate Variables – if the system uses intermediate variables, their definitions are displayed in the output window.
   - Show Name – is a term assigned by the system to the output.





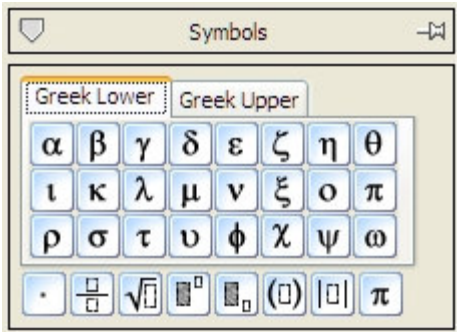
- Show On Diagram – puts the output expression on the diagram when checked.
- Show in Output Window – puts the output expression in the **Output Window** when checked.
- Too Big for Diagram – indicates when the output expression is too big for the diagram window, according to the setting of the slider below.

Maximum Size Allowed On Diagram – slider sets the threshold for the size of expression for all output that appears on the diagram.

Setting the slider to the extreme left (Small) gives the same result as unchecking the Show on Diagram box and will not place the output in the **Diagram Window**.

Setting the slider to the extreme right (Large) will always show the output in the **Diagram Window**.

The Symbols Toolbox



The **Symbols** toolbox lets you easily insert Greek letters into your expressions and constraints. Click the tab to choose from lower case or upper case Greek letters.

The bottom row of buttons in the toolbox lets you insert commonly used math operations.

Inserting Greek Letters

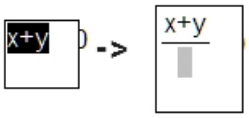
To insert Greek letters into any variable name or expression, click the appropriate tab, **Greek Upper** (upper case letters) or **Greek Lower** (lower case letters), and click the letters to be inserted into the data entry box.

Multiplication and Division Editing Tools

 The **Multiplication** button inserts a multiplication symbol into the expression.

 The **Division** button makes expressions easier to enter and read.

- From the data entry box, enter the numerator of the expression, highlight it,

  and then click **Division**.

  The cursor is then positioned in the denominator.

- If you click the **Division** button first, be sure to place the cursor in the appropriate place before typing the expression.

Square Root Editing Tool

You can enter square roots in one of two ways:

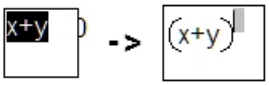
From the data entry box, enter the expression you want inside the square root, highlight the terms, and click the **Square Root** button.

From the data entry box, click the **Square Root** button, then highlight the 0 and type the terms.

Subscript / Superscript Editor

You can enter superscripts or subscripts for variables in one of these ways:

From the data entry box, enter the expression you want sub/superscripted, highlight the terms and click the **Subscript** or **Superscript** button.



- From the data entry box, click the **Sub/Superscript** button and type the values into the grey boxes.

  Note: Make sure the cursor is positioned at the left side of the gray box before typing the sub/superscript.

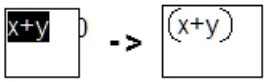
- Another way to make a subscript is to use square brackets –

  A[1] = A1

Parentheses and Absolute Value Notation

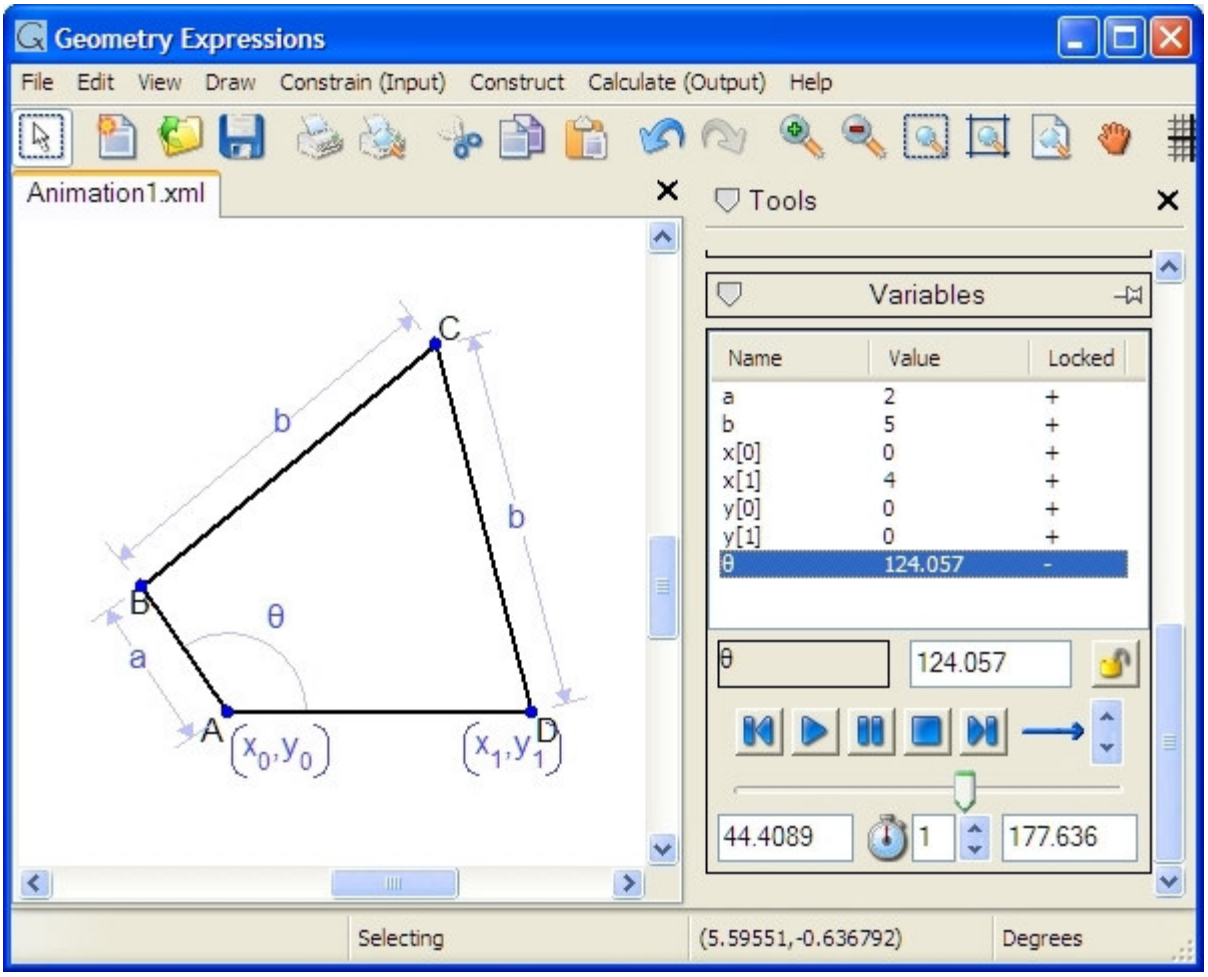
You can add parentheses or an absolute value sign to a term in one of two ways:

From the data input box, type the term(s), highlight it, then click the **Parentheses** or **Absolute** Value button.



From the data entry box, click the **Parentheses** or **Absolute** Value button and enter the terms.

The Variables Toolbox

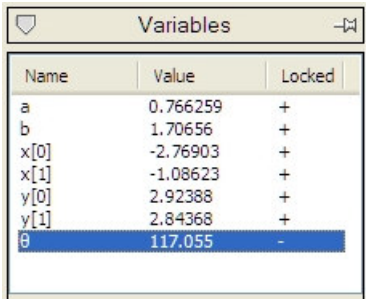


The **Variables** toolbox reports all the variables you have used in the diagram and lets you manipulate their values.

For details on the sections of the toolbox. See:

- Variables list box (p. 118)
- Variable edit window (p. 119)
- Animation console (p. 120)
- Animation mode buttons (p. 121)
- Animation values and duration (p. 121)

Variable List



Variables

Name	Value	Locked
a	0.766259	+
b	1.70656	+
x[0]	-2.76903	+
x[1]	-1.08623	+
y[0]	2.92388	+
y[1]	2.84368	+
θ	117.055	-

This list contains the names of all variables used in your diagram.

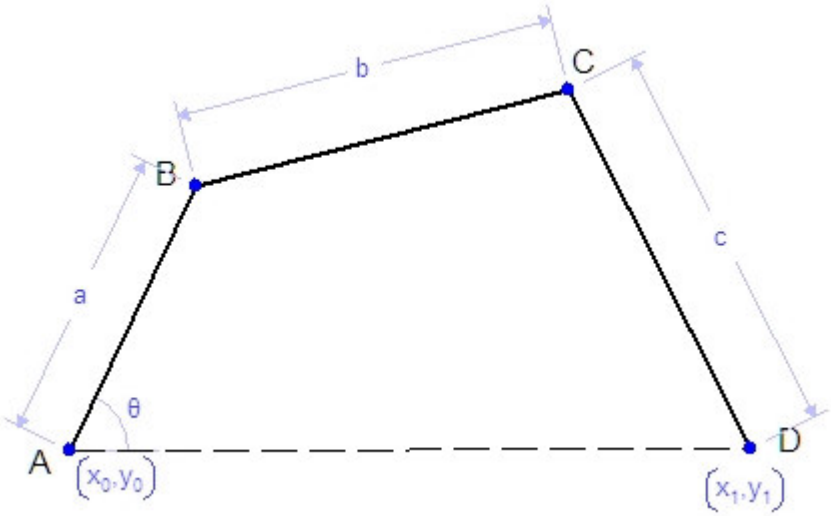
For every variable name, the system shows:

- the current value – these values can be ones that you have explicitly specified, or just taken from the way you sketched the geometry.
- lock status – if the variable is locked (+) its value will not change if you move the geometry or add additional constraints; the unlocked (-) variable is free to change as the geometry moves or changes.

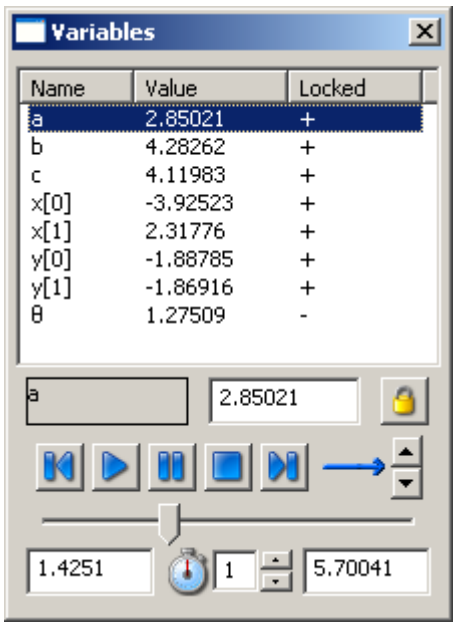
Using the Lock Tool

By default, when you drag points in a Geometry Expressions model, it will adjust the numerical sample values used in the various parameters of the model to accommodate the drag, as best it can.

For example, in the model of a 4 bar linkage below, dragging point B will cause lengths a and b and angle θ to be adjusted appropriately.

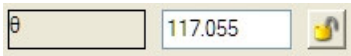


However, you may want the drag to act as if the members AB and BC were rigid, and only angle theta adjustable. To do this you can lock parameters for dragging:



The value of a, for example, can still be set from the Variables panel, but it will not change when the model is dragged.

Changing and Locking the Variable Value



To make a change to the variable list, first click anyplace in the row of the variable you want to change. That row will be highlighted.

<u>To change the value:</u> highlight the value in the edit window and type the new value.

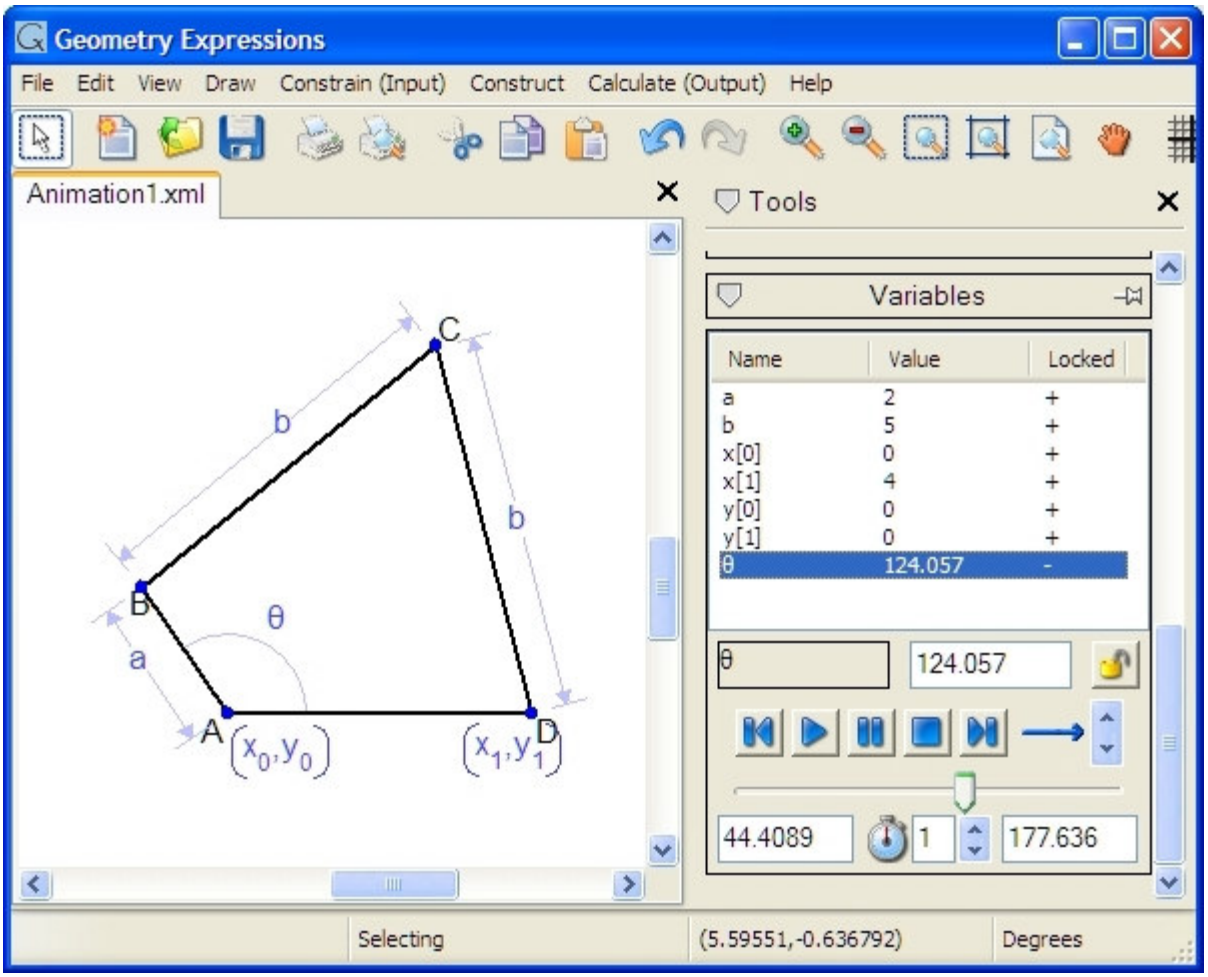
<u>To change the lock status:</u> just click the button –

- to unlock a locked variable
- to lock an open variable

Animation

Your geometry comes to life with the Animation tools. You simply need to select the parameter that drives the animation, give it a range, then Play.

In the diagram below we select θ for the crank of this linkage.



Users of numeric interactive geometry systems may be familiar with the concept of animation based on points animated along line segments or curves. This type of animation can be conveniently modeled in **Geometry Expressions** using the point proportional along a curve constraint along with parameter based animation.

Animation Console


The Animation console works like a standard video console with the **Play**, **Pause**, and **Stop** buttons as well as advance to the **Beginning** and **End** buttons.

Animation Modes

The animation modes can be changed with the up/down arrow buttons. The modes are:

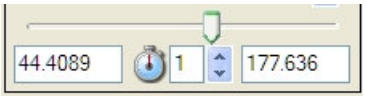
Runs the animation one time through the specified range.

Runs the animation continuously from the beginning to the end of the range.

Runs the animation one time forward and then backward through the specified range.

Runs the animation continuously forward and then backward through the specified range.

Animation Values and Duration



These animation buttons help you adjust the range and speed of the animation.

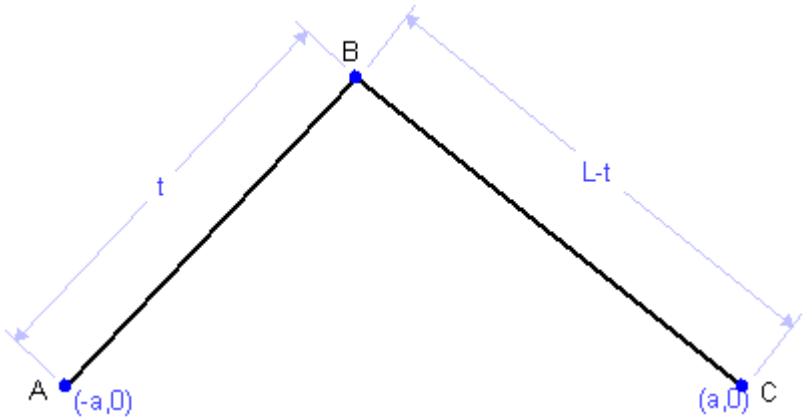
Click and drag the slider along the bar to manually animate the drawing.

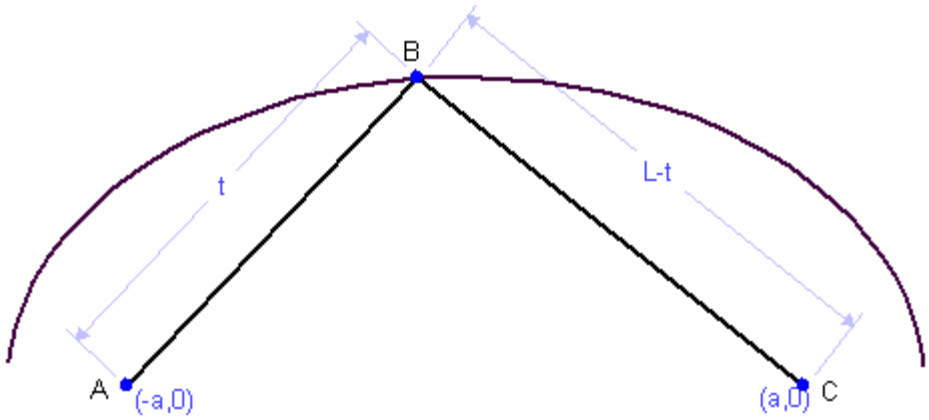
In the two data entry windows at the bottom right and left of the toolbox, specify the range of the animation.

The **Duration** box in the center lets you specify how long the animation takes to play one time through. Values are between 1 and 60 seconds.

Animation and the Locus Tool

Both the construction of the locus and envelope curves, and the animation of the diagram in Geometry Expressions can be defined in terms of any variable. For example in the model below, we can create a locus over values of the variable t (other variables will be kept constant).




B
t
L-t
A
(-a,0)
(a,0)
C

File Menu

The **File** menu contains the standard Windows file handling operations with options for copying and exporting to other programs. Several of the options are also available from the icon bar.

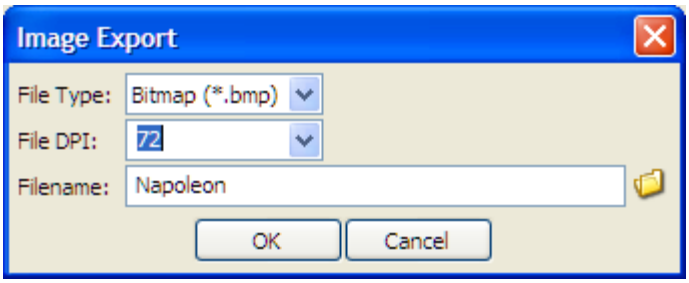
Menu Option	Function
New	Creates a new project.
Open. . .	Brings up the Open File dialog box so you can open a project.
Close	Closes the current file or, if multiple files are open, the file on top
Save	Saves the file. If you have not yet saved the current work to a file, the **Save File As** dialog box lets you specify where to save the project file.
Save As . . .	Brings up the **Save File As** dialog box so you can specify where to save the project file or change the file name
Export	Export the file as Windows Metafile (.emf) or an image file
Page Setup. . .	Displays the page set-up dialog box for choosing a printer and print options
Print Preview	Displays the printout by pages
Print. . .	Displays the standard system Print dialog
Recent files	Click to display a list of the most recently used files. Selecting one opens it.
Exit	Exits the program, after prompting for save

Exporting a Drawing

To export your drawing to another program, choose

File > **Export**. You can export the drawing as a *Windows Enhanced Metafile* or in one of several standard image formats.

File > **Export** > **Image File** displays the **Image Export** dialog.



- Click the right hand arrow to select one of the following formats:

  .bmp .jpg

  .tiff .png

  .xpm
- Set the resolution in the File DPI box - click the down arrow and select the appropriate setting, or enter a custom setting.
- Enter the Filename or click the folder icon to select the appropriate folder and file.
- Click OK to save or Cancel to abort the save.

Edit Menu

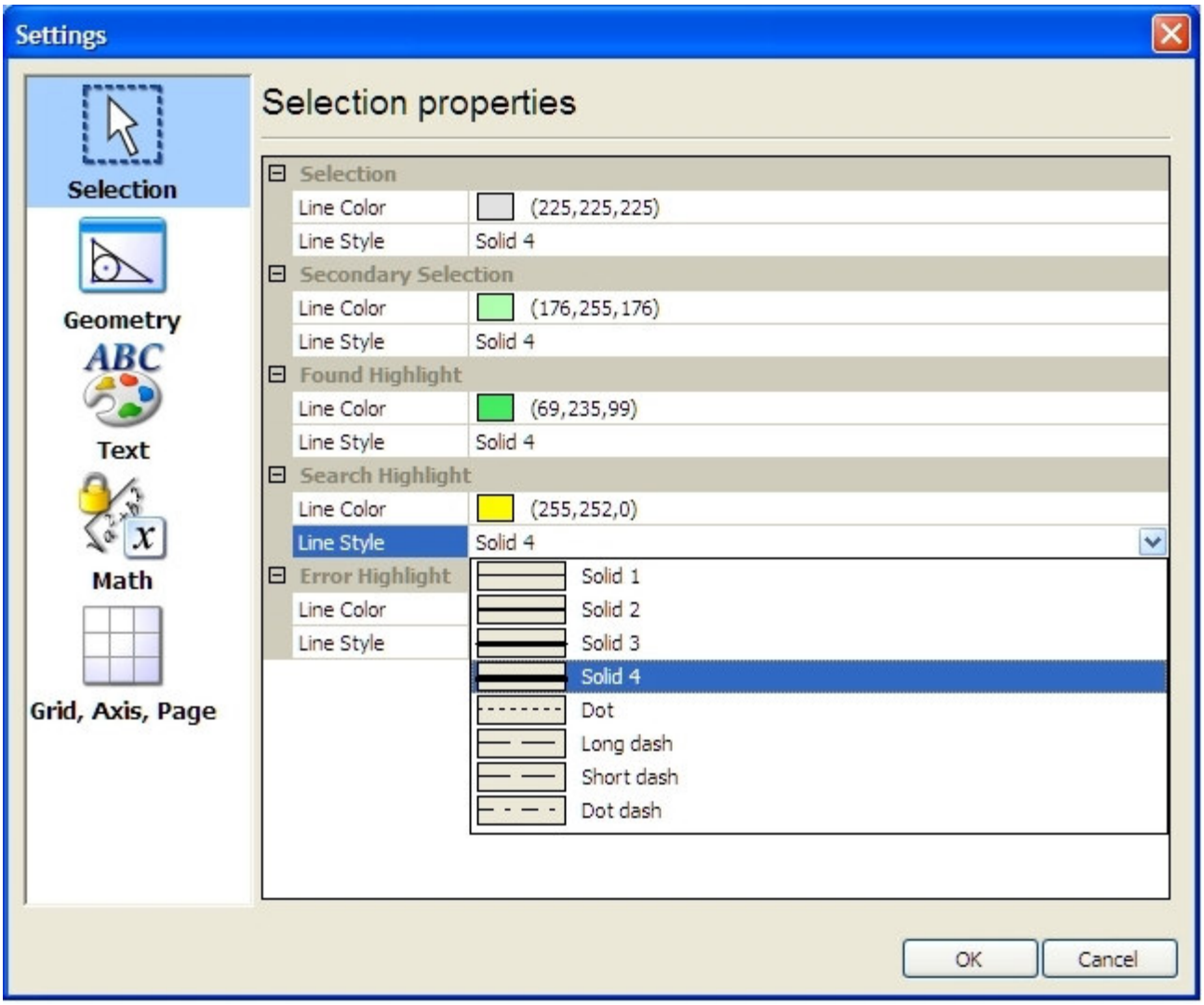
The **Edit** menu contains the standard Windows editing operations as well as ways of dealing with constraint conflicts and all of the program settings. Several of the options are also available from the icon bar.

Menu Option	Function	When Available
Undo	Reverses actions starting with the last one.	After any action has been taken.
Redo	Reinstates actions starting with the last one that was undone.	After using **Undo**
Select	When checked, the select mode is active	Select mode is always active except when using a Drawing tool or moving or panning the drawing.
Select All	Selects everything in the drawing window	Always
Cut	Deletes an object, but saves it so it can be pasted somewhere else.	An object is selected
Copy	Does not delete the object, but saves is so it can be pasted somewhere else.	An object is selected
Copy As	Displays a submenu of choices for copying mathematics into other programs	An expression or formula is selected
Copy Drawing	Copies everything in the drawing window as an Enhanced Metafile (.emf)	Always
Paste	Puts whatever has been cut or copied into the current document	Object(s) cut or copied
Delete	Deletes whatever is selected, without saving it.	One or more object(s) selected
Convert to Constraint	Tries to convert a selected output [$\Rightarrow 2\theta$] to an input constraint if there is no conflict with other constraints	An output expression is selected
Convert to Measurement	Converts the selected constraint into a measurement (output)	An input constraint is selected
Properties. . .	Lets you edit the display properties of the selected object(s)	One or more objects of the same type are selected

Settings. . .	Sets the default appearance and properties for the project's drawing, text and mathematics	Always

Adjusting the Drawing's Default Settings

The default settings are grouped by type, listed on the left side of the Settings dialog.



Selection - set the line color and style for each selection type.

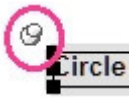
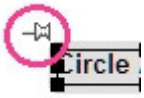
Geometry - set font related properties for labels; color and size / style for other geometric elements.

Text - set font properties and the Pinned state. Pinned **Text** can not be moved relative to the **Page Boundaries**.

Math - set the properties for alpha-numeric input and output; mathematical calculation defaults

Grid, Axis, Page - set properties of the Major and Minor Grid, the coordinate Axes, and the Page Boundary lines.

To see the possible values for each property, click the row. An icon will appear at the right end of the row (except the Point Size selection under the Font property – you can enter the point size directly). Click the icon to display the selection dialog or drop-down menu of choices .

Changing a default for a type of drawing entity will apply to all entities of that type except ones whose properties have been individually set, by selecting it and then choosing **Edit** > **Properties** or right clicking **Properties** from the context menu. Also, text or expressions that were individually pinned  or unpinned  will not be affected by changes to the default Pinned settings.

View Menu

The **View** menu has **Zoom** operations pertaining to the screen view, and **Scale** operations pertaining to the page view.

The table below lists the complete summary of **View** functions.

Menu Option	Function	When Available
Hide	Hides a selection	One or more element(s) are selected.
Show all	Displays any entities that were hidden	One or more element(s) are hidden.
Zoom In	Makes the drawing details larger without affecting the size on the printed page. (The text gets larger on the screen.)	Always available – (most useful when there is something in the drawing window).
Zoom Out	Makes the drawing details smaller without affecting the size on the printed page. (The text gets smaller on the screen.)	Always available – (most useful when there is something in the drawing window).
Zoom To Selection	Lets you make a selection and adjusts it to fit the drawing window. (The text gets larger on the screen.)	Always available – (most useful when there is something in the drawing window).
Zoom To Fit	The entire diagram is displayed in the drawing window. (The text size changes with the geometry.)	Always available – (most useful when there is something in the drawing window).
Zoom To Page	The whole page is displayed in the drawing window. (The text size changes with the geometry.)	Always available – (most useful when there is something in the drawing window).
Pan View	Allows you to move the contents of the drawing window without changing its position on the page.	Always available – (most useful when there is something in the drawing window).
Scale Geometry Up	Enlarges only the geometry. (The text size on the screen doesn't change.)	Always
Scale Geometry Down	Shrinks only the geometry. (The text size on the screen doesn't change.)	Always

Scale Geometry To Selection	Lets you select a portion of the geometry and adjusts it to fit the drawing window (The text size on the screen doesn't change.)	Always
Scale Geometry To Fit	Adjusts all geometry to fit in the drawing window. (The text size on the screen doesn't change.)	Always
Scale Geometry To Page	Adjusts all geometry to fit inside the specified page boundaries. (The text size doesn't change relative to the page.)	Always
Move Geometry	When checked, click and drag to move the drawing contents with respect to the page boundaries.	Always. (Make sure **Page Boundaries** is checked to see the results.)
Axes	When checked, the axes are displayed.	Always
Grid	When checked, the grid is displayed.	Always
Page Boundaries	When checked, the page boundaries are displayed.	Always
Tools	When checked, the toolboxes are displayed to the side of the drawing window.	Always
Output	When checked, the output window is displayed.	Always

Checked menu options are toggles:

- Checked indicates the option / mode is active or displayed.
- Unchecked indicates the option / mode is inactive or hidden.

Click the selection to change its state.

Toolbox Menus

The menus with the same name as the toolboxes at the side of the screen just give another way of accessing the same functions.

Menu Option	**Selection**		
Draw	Point	Vector	Text
	Line Segment	Polygon	Expression
	Infinite Line	Circle	
Constrain	Distance / Length	Direction	Tangent
	Radius	Slope	Incident
	Perpendicular	Coordinate	Congruent
	Angle	Coefficients	Parallel
	Equation	Proportional	
Construct	Midpoint	Parallel	Reflection
	Intersection	Perpendicular	Translation
	Perpendicular Bisector	Tangent to Curve	Rotation
	Angle Bisector	Polygon	Dilation
	Locus		
Calculate	Distance / Length	Coordinates	Slope
	Direction	Coefficients	Angle
	Radius	Area	Perimeter

Help Menu

The **Help** menu lets you access the help system, check for updates, change the program's language, and gives you information about the program's license and version. The menu selections are always available.

Menu Option	**Function**
Dynamic Help	Invokes the Help system
Contents. . .	Look in the Table of Contents; add new or refer to saved bookmarks.
Index. . .	Look in the **Help** index. There is also a facility to Search index headings.
Search. . .	Search the **Help** topics for keywords.
Language	The current version of Geometry Expressions can be displayed in **English**, **French**, **German**, or **Spanish**. Choose one and restart the program.
License. . .	Displays information about your license.
Check for Updates. . .	Prompts you to save your work, checks for new versions of Geometry Expressions, then restarts the program.
About. . .	Contains the current version of the program, the copyright notice, and the link to Geometry Expressions' website.

Context Menus

Context Menus pop up when you right-click with the cursor positioned anywhere in the drawing window.

- The general context menu – appears when you right click and nothing is selected.
- The selection context menu – appears when one or more elements in the drawing window are selected. Some menu entries my be inactive, depending on which elements are selected.

The General Context Menu

Right-click anywhere in the Drawing Window to display a context menu. If nothing in the window is selected, the menu choices are the following:

Menu Option	Function	When Available
[file name]	Displays the name of the current file. If you have not specified one, the default is displayed – "unnamedX".	Always
Close	Closes the current file	Always
Save	Updates a file that already exists	The file has been saved
Save As. . .	Saves a file for the first time and prompt for the filename and path	Always
Select All	Selects everything in the drawing window	Always
Copy Drawing	Copies everything in the drawing window as an Enhanced Metafile (.emf)	Always
Paste	Puts whatever has been cut or copied into the current document	Object(s) cut or copied
Show All	Displays any entities that were hidden	One or more element(s) are hidden

Selection Context Menu

Menu Option	Function	When Available
Selection	Indicates an item is selected	Always
Cut	Deletes an object, but saves it so it can be pasted somewhere else.	An object is selected
Copy	Does not delete the object, but saves is so it can be pasted somewhere else.	An object is selected
Copy As	Displays a submenu of choices for copying mathematics into other programs	An expression or formula is selected
Paste	Puts whatever has been cut or copied into the current document	Object(s) cut or copied
Delete	Deletes whatever is selected, without saving it.	One or more object(s) selected
Constrain (Input)	Displays a submenu identical to the one in the same drop-down menu on the **Menu Bar**	A geometry object is selected
Construct	Displays a submenu identical to the one in the same drop-down menu on the **Menu Bar**	A geometry object is selected
Calculate (Output)	Displays a submenu identical to the one in the same drop-down menu on the **Menu Bar**	A geometry object is selected
Hide	Hides one or more selections	One or more object(s) selected
Properties. . .	Lets you edit the display properties of the selected object(s)	One or several similar objects are selected
[Convert to Calculation (Output)]	Deletes the selected constraint and calculates the equivalent output	Only displayed when a constraint is selected
[Convert to Constraint (Input)]	Changes the calculation to an input constraint	Only displayed when an output expression is selected

Index